Appendix A Phase transition test models

This appendix presents details on the test models used for validation. Python implementations of both models are available on the author's Github: https://github.com/dylewsky/phase_transition_EWS

Water-vegetation model

This model, presented in [1], describes a two-dimensional lattice on which each site has nonnegative values for water quantity w_{ij} and biomass B_{ij}. Stochastic dynamics are defined as follows:

$$\begin{aligned} \frac{dw_{i,j}}{dt} =& R - w_{i,j} - \lambda w_{i,j} B_{i,j} \\ &+ D\left(w_{i+1,j} + w_{i,j+1} + w_{i-1,j} + w_{i,j-1} - 4w_{i,j}\right) + \sigma_w dW_{i,j} \\ \frac{dB_{i,j}}{dt} =& \rho B_{i,j}\left(w_{i,j} - \frac{B_{i,j}}{B_c}\right) - \mu \frac{B_{i,j}}{B_{i,j} + B_O} \\ &+ D\left(B_{i+1,j} + B_{i,j+1} + B_{i-1,j} + B_{i,j-1} - 4B_{i,j}\right) + \sigma_B dW_{i,j} \end{aligned} \tag{A1}$$

$dW_{i,j}$ represents stochastic white noise of unit variance. Values for constants are set to:

$$\begin{aligned} D &= 0.05 \\ \lambda &= 0.12 \\ \rho &= 1 \\ B_c &= 1 \\ \mu &= 2 \\ B_O &= 1 \\ \sigma_w &= 0.025 \\ \sigma_B &= 0.0625 \end{aligned} \tag{A2}$$

Global rainfall R is varied during the simulations. When it falls below $R = R_c \approx 1.936$, a phase transition is observed in which the system rapidly loses biomass and shifts to a stable desert equilibrium. Simulations are carried out on a 200×200 square lattice with periodic boundary conditions and randomized starting and ending values between which R is linearly interpolated (passing through R_c in transition runs, held far from R_c in null runs).

Sea ice percolation model

This is a dynamical model constructed to serve as a toy example of a percolation phase transition in sea ice. Water diffuses through a cubic lattice of cells, the boundaries between which are either closed or open. The percolation transition occurs when the fraction of open boundaries exceeds some threshold, so we define a tunable temperature parameter which mediates the melting (opening) and refreezing (closing) rates of the boundaries in order to vary this ratio. At the theoretically-established critical threshold (boundary closure fraction $p_c \approx 0.25$ in 3D), the permeation of water through the lattice undergoes an abrupt regime change. Simulations are carried out on a $256 \times 256 \times 64$ cubic lattice with periodic boundary conditions.

References

[1] Dakos, V., Kéfi, S., Rietkerk, M., van Nes, E.H., Scheffer, M.: Slowing down in spatially patterned ecosystems at the brink of collapse vasilis. American Naturalist **177**(6) (2011). https://doi.org/10.1086/659945