Improving environment drives dynamical change in social game structure

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Supplementary Materials

Supplementary Text

In the main text, we adopted a simple form $p(E, n_c) = E^{\delta^{n_c - 1}}$ for the success probability. In order to show that the results are robust and general, here we present the corresponding results for another functional forms. The necessary conditions for the success probability $p(E, n_c)$ are as follows: it is a monotonically increasing function of the number of cooperators n_c and environmental value E, and moreover $p(0, n_c) = 0$ and $p(1, n_c) = 1$.

Model S1

As in the main text, we assume an exponential function of E,

$$p(E, n_c) = E^{\epsilon}.$$

According to the above conditions, the exponent ϵ should be a decreasing function of n_c. Let it vary between $1/\delta$ and δ, where δ is an arbitrary small number ($0 \leq \delta \leq 1$). In other words, the smaller δ is, the stronger the n_c dependence of $p(E, n_c)$.

For instance, we may write

$$\epsilon = \cfrac{1}{x + \cfrac{1}{x + \frac{1}{\delta} - \delta}} + \delta,$$

where

$$x = tan\left(\frac{\pi n_c}{4}\right)$$

is an increasing function of n_c. Note that ϵ varies from $1/\delta$ to δ as n_c changes from 0 to 2. First, we assume $\delta = 1/3$ for figures S1 and S2. In figure S1*a*, the probability $p(E, n_c)$ is plotted against n_c for $E = 0, 0.25, 0.5$ and 0.75. In figure S1*b*, the probabilities for $n_c = 0, 1$ and 2 are shown on a line of unit length for $E = 0, 0.25, 0.5, 0.75$ and 1. This figure S1 should be compared with figure 2 in the main text. Figure S2*a* and *b* show the trajectories in the Δp_1-Δp_2 plane for $c/b = 1/2$ and $1/3$, respectively. Figure S2*c* shows the trajectories in the E-c/b plane for $c/b = 1/2$ (solid arrows) and $1/3$ (dashed arrows). The figure S2 should be compared with figure 3 in the main text. It is remarked that the course of change in game structure is

essentially the same. Similarly, the panels in figures S3 and S4 are drawn for $\delta = 1/5$.

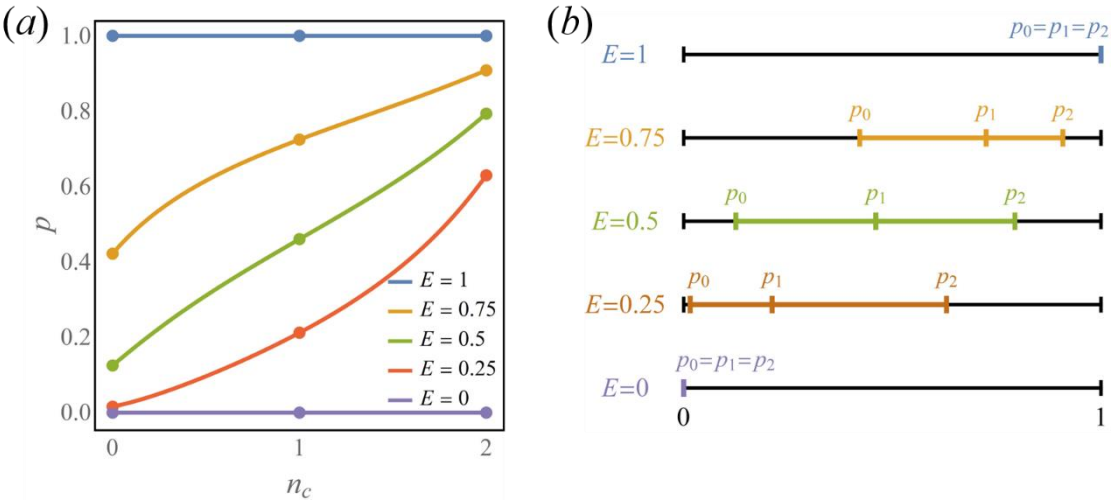
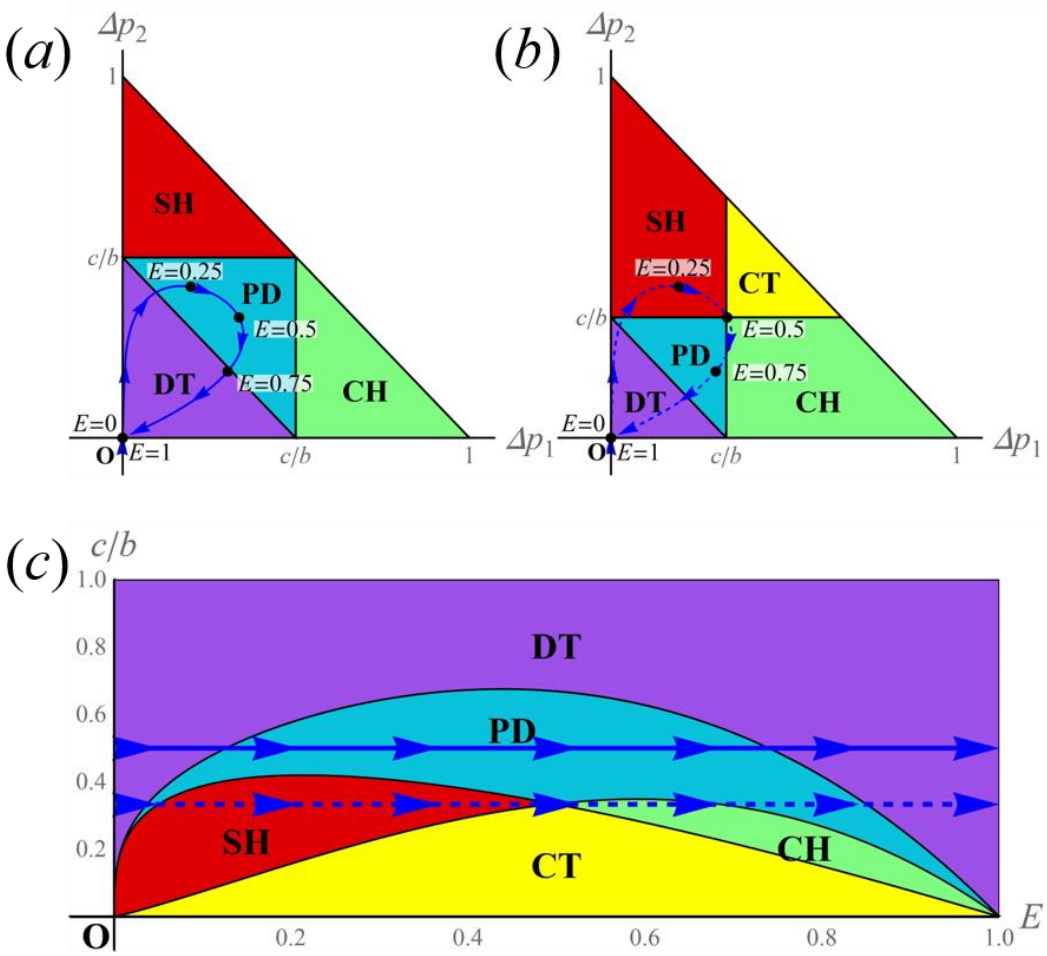


Figure S1



$\delta = 1/3$

Figure S2

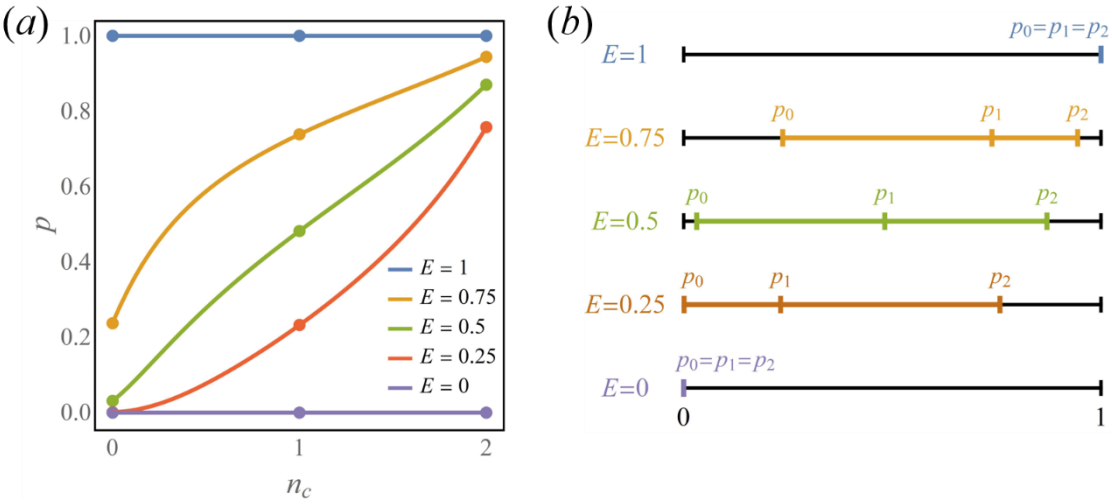
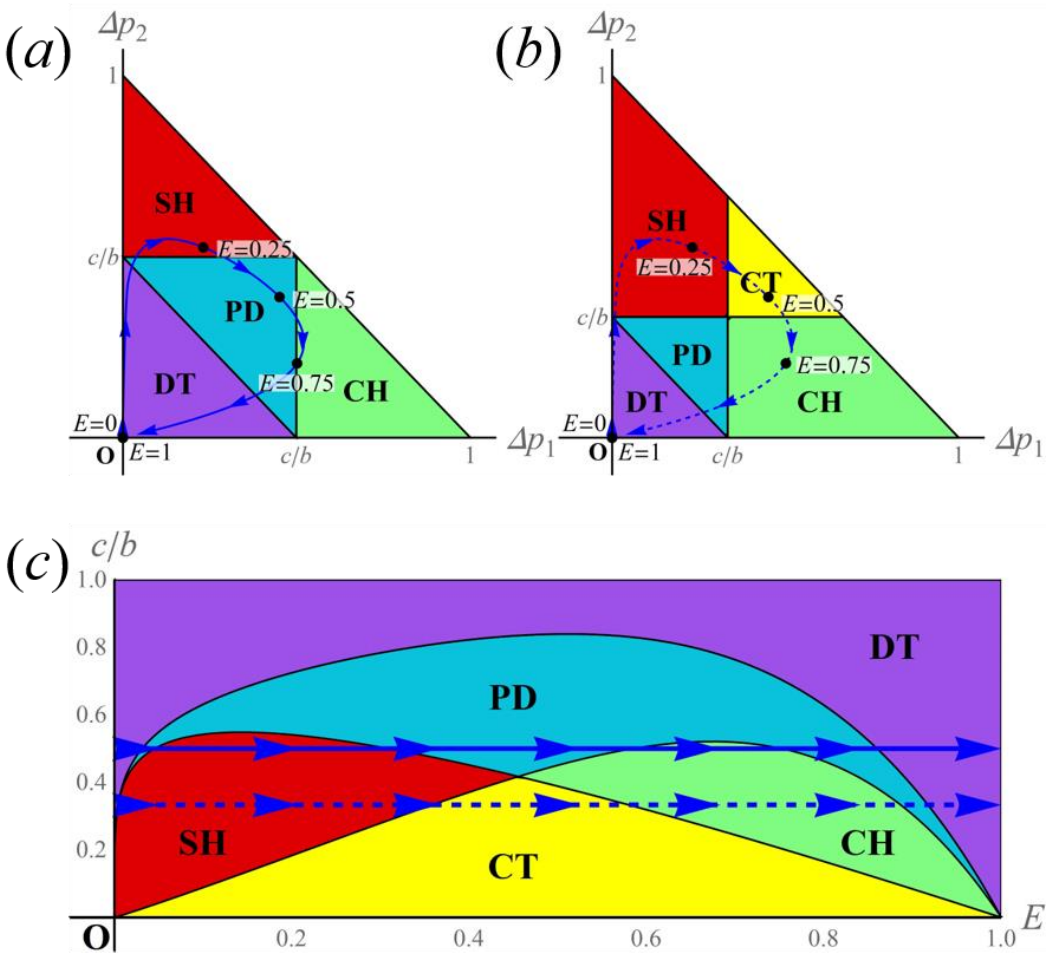


Figure S3



$\delta = 1/5$

Figure S4

Model S2

As another example, let us use

$$p(E, n_c) = \sqrt{E}\left(\frac{1 + n_c}{3}\right)^{2(1-E)}.$$

In figure S5*a*, the probability $p(E, n_c)$ is plotted against n_c for $E = 0, 0.25, 0.5$ and 0.75. In figure S5*b*, the values of $p(E, n_c)$ for $n_c = 0, 1$ and 2 are shown on a line of unit length for $E = 0, 0.25, 0.5, 0.75$ and 1.

Figures S6*a* and *b* show the trajectories in the Δp_1-Δp_2 plane for $c/b = 1/3$ and $1/5$, respectively. Figure S6*c* shows the trajectories in the E-c/b plane for $c/b = 1/3$ (solid arrows) and $1/5$ (dashed arrows).

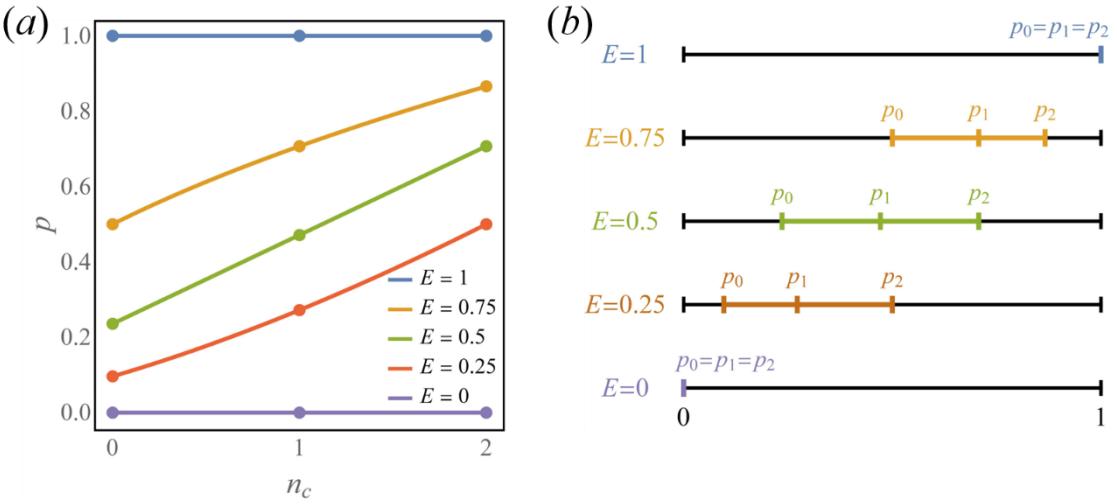


Figure S5

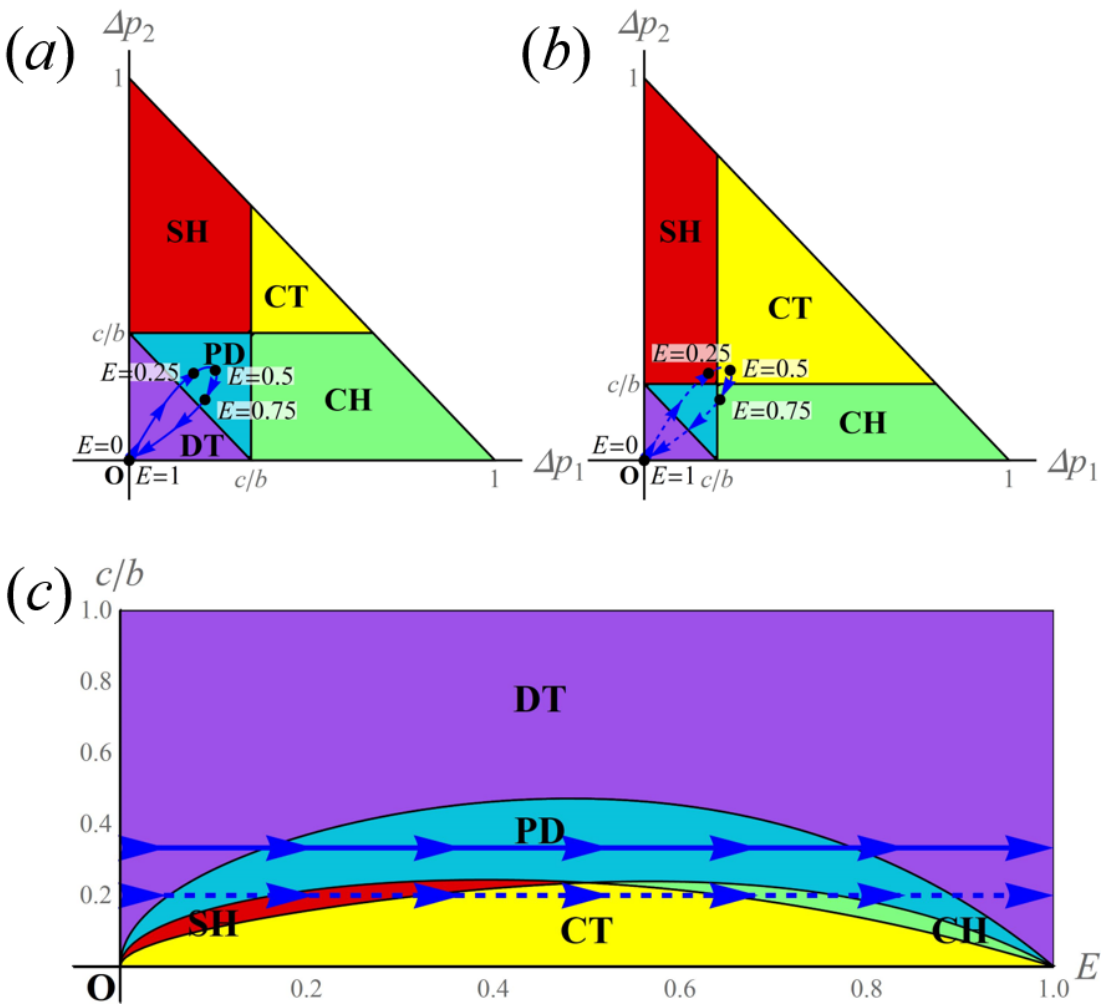


Figure S6