

## ONLINE APPENDIX A

### Resource availability set by the resident consumer species in a seasonally varying environment

Equation (3) specifies the conditions under which a second consumer can invade a community consisting of a resource species and a resident consumer species:

$$\frac{\frac{1}{\tau} \int_0^\tau e_I a_I(T(t)) R_{C_R}(T(t)) dt}{\frac{1}{\tau} \int_0^\tau d_I(T(t)) dt} > 1 \quad (\text{A.1})$$

where  $a_I(T(t))$ ,  $d_I(T(t))$  and  $e_I$  depict, respectively, the invader's per capita consumption and mortality rates and its conversion efficiency, and  $R_{C_R}(T(t))$  is the instantaneous resource abundance set by the resident consumer when it is at a stationary state with the resource in the absence of the invader. Evaluating the invasion criterion requires that we specify  $R_{C_R}(T(t))$ .

We know from basic consumer-resource theory that the resource availability set by a consumer is a function of its birth rate (conversion efficiency times the attack rate) and mortality rate (Murdoch et al., 2003). In a constant environment,  $R_{C_R}(T) = \frac{d_R(T)}{e_R a_R(T)}$ , where  $\frac{d_R(T)}{e_R a_R(T)}$  is the consumer's  $R^*$ , i.e., the steady state resource abundance at which the consumer's birth and mortality rates are exactly balanced. If a steady-state consisting of the resource and the resident consumer exists in a seasonally varying environment, we would expect it to constitute a stationary distribution of abundances, i.e., an annual abundance pattern that repeats itself year after year. The resource species' stationary state would then constitute a set of values of  $R_{C_R}(T(t))$  corresponding to each time point  $t$  during the year.

We use an argument inspired by Armstrong and McGehee (1980) to show that when seasonal variation occurs in a predictable manner (i.e., the mean annual temperature and the amplitude of fluctuations remain constant over time) and when the consumers exhibit linear (Type I)

functional responses,  $R_{C_R}(T(t))$  can be approximated by  $\frac{d_R(T(t))}{e_R a_R(T(t))}$ . The argument is as follows.

Since the resource abundance set by the resident consumer is determined by its birth and death rates regardless of whether the environment is constant or time-varying, the steady state resource abundance at any time  $t$  (i.e.,  $R_{C_R}(T(t))$ ) should also be determined by the resident consumer's birth ( $e_R a_R(T(t))$ ) and death rate ( $d_R(T(t))$ ) at time  $t$ . The temperature response functions  $a_R(T(t))$  and  $d_R(T(t))$ , which are mechanistically derived based on the underlying biochemical processes, are continuous functions of temperature. When temperature varies seasonally with a constant mean and amplitude,  $a_R(T(t))$  and  $d_R(T(t))$  become continuous functions of time. Since consumer-resource dynamics are described by ordinary differential equations, resource abundance  $R(t)$  is also a continuous function of time.

Because seasonal variation is sinusoidal with a constant period  $\tau = 365$  days, the time-varying parameters  $a_R(T(t))$  and  $d_R(T(t))$  return to their original values after a time period  $\tau$  with an amplitude determined solely by the amplitude of seasonal fluctuations (Fig. A1(a)-(b)). Similarly, since seasonal variation is the only source of periodicity in consumer-resource dynamics, as would be the case when the consumer has a Type I functional response, resource and consumer abundances also return to their original values after a period of 365 days (Fig. A1(c)-(d)).

Let  $\frac{dC(t)}{dt} = C f(R(t))$  where  $f(R(t)) = \frac{1}{C} \frac{dC(t)}{dt} = e_R a_R(T(t)) R(t) - d_R(T(t))$  is the resident consumer's per capita growth rate. In a seasonally varying environment, the average per capita growth rate over the year is given by  $\overline{f(R(t))} = \frac{1}{\tau} \int_0^\tau \frac{1}{C} \frac{dC(t)}{dt} dt = \frac{1}{\tau} \int_0^\tau f(R(t)) dt$ . Since consumer abundance returns to its original value after time  $\tau$ ,  $\int_0^\tau \frac{1}{C} \frac{dC(t)}{dt} = \ln C(\tau) - \ln C(0) = 0$ . Therefore,  $\overline{f(R(t))} = 0$ .

Note that  $\overline{f(R(t))} = \frac{1}{\tau} \int_0^\tau e_R a_R(T(t)) R(t) - d_R(T(t)) dt$ , which we can rewrite as:

$$\int_0^\tau e_R a_R(T(t)) R(t) dt = \int_0^\tau d_R(T(t)) dt. \quad (\text{A.2})$$

From Equation (A.2) we can see that  $\overline{f(R(t))} = 0$  when  $R(t) = \frac{d_R(T(t))}{e_R a_R(T(t))}$ . Mathematically speaking,  $R(t) = \frac{d_R(T(t))}{e_R a_R(T(t))}$  is not a unique solution because the equality of integrands is not necessary for Equation (A.2) to hold true (i.e.,  $\overline{f(R(t))}$  can be equal to zero without  $R(t) = \frac{d_R(T(t))}{e_R a_R(T(t))}$ ). In order for  $R(t) = \frac{d_R(T(t))}{e_R a_R(T(t))}$  to be a unique solution,  $R(t) = \frac{d_R(T(t))}{e_R a_R(T(t))}$  has to be true for all  $\tau$ . The biological basis of our question helps narrow down the scope of this problem. We are investigating latitudinal directionality in invasion success. Seasonal temperature variation with a period of  $\tau = 365$  days is, therefore, the thermal regime of relevance to our question, and the functional forms for  $a_R(T(t))$  and  $d(T(t))$  we use in our model are the empirically quantified thermal reaction norms for attack and mortality rates that have evolved in response to seasonal temperature variation (Savage et al., 2004; Englund et al., 2011; Amarasekare and Johnson, 2017).

Because we do not know the functional forms of  $a_R(T(t))$  and  $d(T(t))$  in thermal environments other than seasonal variation, we cannot determine whether the equality of integrands holds for all  $\tau$ . However, we can verify whether  $R(t) = \frac{d_R(T(t))}{e_R a_R(T(t))}$  is a valid solution when  $\tau = 365$ . We use the full dynamical model of the pairwise consumer interaction to determine whether  $R(t) = \frac{d_R(T(t))}{e_R a_R(T(t))}$  ( $t = 0, \dots, 365$ ) represents the distribution of resource abundances corresponding to the stationary state attained by the resource and the resident consumer in a seasonally varying thermal environment.

The dynamics of the pairwise interaction between the resource and the resident consumer are given by:

$$\begin{aligned}\frac{dR(t)}{dt} &= b(T(t))R(t)\left(1 - q(T(t))R(t)\right) - d(T(t))R(t) - a_R(T(t))R(t)C_R(t) \\ \frac{dC_R(t)}{dt} &= e_R a_R(T(t))R(t)C_R(t) - d_R(T(t))C_R(t)\end{aligned}\tag{A.3}$$

When we numerically integrate Equation (A.3) under seasonal variation, the resource-consumer interaction attains a stationary distribution of abundances in the form of a limit cycle with a period of one year. The reason for this is as follows. A pairwise consumer-resource interaction in which the resource has logistic growth and the consumer has a Type 1 functional response reaches a steady state with damped oscillations in a constant environment. Since seasonal forcing is the only form of temporal variation in the system and there are no intrinsically generated consumer-resource oscillations (such as would occur if the consumer had a saturating functional response), the system attains a stationary distribution of abundances in which the species' abundances return to their original values after a period of one year. As shown in Fig. A1(c)-(d),  $\frac{d_R(T(t))}{e_R a_R(T(t))}$  provides an accurate approximation of the instantaneous resource abundance  $R_{C_R}(T(t))$  in this limit cycle solution. In general, this approximation works well when the resident consumer is adapted to exploit the resource in the seasonal thermal regime they experience in common, the situation pertinent to our question of directionality in invasion success. This occurs when the maximum resource birth rate ( $b_{T_{opt}}$ ) is sufficiently high and the response breadth ( $s_b$ ) is sufficiently wide that the consumer can persist on the resource (Fig. A2(a)), the consumer's intrinsic mortality rate ( $d_{R_{T_R}}$ ) is high enough to prevent resource overexploitation (Fig. A2(b)), the consumer's temperature sensitivities of mortality above the low temperature threshold does not exceed those of the resource ( $A_{d_R} \leq A_d$ ; Fig. A2(c)), and when the consumer's response breadth ( $s_{a_R}$ ) is sufficiently wide relative to that of the resource that it can exploit the resource when it is available but not so wide as to overexploit it ( $\frac{s_{a_R}}{s_b} \geq 1$ ), and the consumer's attack rate optimum  $a_{RT_{opt}}$  increases (decreases) as its response breadth decreases (increases) such that the consumer is able to persist on the resource without overexploiting it (Fig. A2(d)).

In summary, the match between the limit cycle solution to Equation (A.3) ( $R_{C_R}(T(t))$ ) and  $\frac{d_R(T(t))}{e_R a_R(T(t))}$  means when the resource and resident consumer reach a stationary state in a seasonally

varying environment,  $R_{C_R}(T(t))$ , the stationary resource abundance at time  $t$  ( $t = 0, \dots, 365$ ) is well-approximated by  $\frac{d_R(T(t))}{e_R a_R(T(t))}$ . By substituting  $R_{C_R}(T(t)) = \frac{d_R(T(t))}{e_R a_R(T(t))}$  in Equation (3) in the main text, we can obtain a complete analytical expression for the invasion criterion (Equation (4) in the main text).

## ONLINE APPENDIX B

### Temperature responses of species' traits

#### Temperature response of mortality

In all ectotherms, density-independent per capita mortality rate increases with temperature (Savage et al. (2004) and references in Gillooly et al. (2001, 2002)) within which temperature range the underlying biochemical processes are fully functional and reproduction and development can occur (Johnson and Lewin, 1946; Sharpe and DeMichele, 1977; Schoolfield et al., 1981; Ratkowsky et al., 2005). Below this range, mortality increases with decreasing temperature due to the freezing of body fluids and other related phenomena (Savage et al. (2004) and references in Gillooly et al. (2001, 2002); see Fig. 1k and l). The complete mortality response can be described by the following modification to the Boltzmann-Arrhenius function for reaction kinetics:

$$d_X(T) = d_{X_{T_R}} e^{A_{d_X} \left( \frac{1}{T_R} - \frac{1}{T} \right)} \left( 1 + e^{A_L \left( \frac{1}{T_L} - \frac{1}{T} \right)} \right) \quad (\text{B.1})$$

where  $d_X(T)$   $X = R, I$  is the mortality rate at temperature  $T$  (in  $K$ ),  $A_{d_X}$  is the Arrhenius constant, which quantifies how fast the mortality rate increases with increasing temperature,  $T_{R_X}$  is a reference (baseline) temperature at which mortality is equal to  $d_{X_{T_R}}$ . The reference temperature occurs within the range where enzymes are 100% active (typically between  $20-30^\circ C$ ,  $24-25^\circ C$  being the most common; Johnson and Lewin (1946); Sharpe and DeMichele (1977); Schoolfield et al. (1981); Ratkowsky et al. (2005)). The parameter  $T_{L_X}$  is the temperature

threshold at which mortality starts to increase with decreasing temperature, and  $A_{L_X}$  quantifies how quickly the mortality rate decreases with decreasing temperature. Note that  $A_{d_X} > 0$  and  $A_{L_X} < 0$ .

### Temperature response of birth and consumption rates

A large number of studies spanning a range of ectothermic taxa show that per capita birth and consumption rates exhibit unimodal responses to temperature (Dreyer and Baumgartner, 1996; Carriere and Boivin, 1997; Morgan et al., 2001; Jandricic et al., 2010; Hou and Weng, 2010; Dannon et al., 2010; Dell et al., 2011; Englund et al., 2011; Amarasekare and Savage, 2012; Amarasekare, 2015). Both are well-described by a Gaussian function:

$$A_X(T) = A_{X_{\text{Topt}}} e^{-\frac{(T-T_{\text{opt}_{A_X}})^2}{2s_{A_X}^2}} \quad (\text{B.2})$$

where  $T_{\text{opt}_{A_X}}$  ( $A = a, b, X = R, I$ ) is the temperature at which the birth (consumption) rate is maximal ( $A_{X_{\text{Topt}}}$ ), and  $s_{A_X}$  determines how fast or slowly the response decays from the optimum (Fig. 1a and b). It provides a statistically quantifiable index of the response breadth, i.e., the temperature range over which the species can reproduce and exploit resources.

### Temperature response of resource self-limitation

Experiments on insects suggest that the temperature response of the self-limitation (i.e., the per capita intra-specific coefficient ( $q(T)$ )) can be monotonic or unimodal (Amarasekare and Coutinho, 2014; Amarasekare, 2015; Johnson et al., 2015)). When self-limitation strength increases with increasing temperature, as is the case when increasing activity levels increase the per individual demand for resources,  $q(T)$  is given by the Boltzmann-Arrhenius relationship:

$$q(T) = q_{T_R} e^{A_q \left( \frac{1}{T_R} - \frac{1}{T} \right)} \quad (\text{B.3})$$

where  $q(T)$  is the self-limitation strength at temperature  $T$ ,  $A_q$  is the Arrhenius constant,  $T_R$  is the reference temperature as described above, and  $q_{T_R}$ , the self-limitation strength at the reference temperature.

When self-limitation is strongest at temperatures optimal for reproduction, as is the case when the demand for resources is most intense during periods of peak reproductive activity (Amarasekare and Coutinho, 2014; Amarasekare, 2015; Johnson et al., 2015),  $q(T)$  is unimodal and well-described by a Gaussian function:

$$q(T) = q_{T_{\text{opt}_q}} e^{-\frac{(T-T_{\text{opt}_q})^2}{2s_q^2}} \quad (\text{B.4})$$

where  $T_{\text{opt}_q}$  is the temperature at which self-limitation is the strongest ( $q_{T_{\text{opt}_q}}$ ), and  $s_q$  depicts the the temperature range over which self-limitation operates. We use the Gaussian form (Equation (B.4)) in our analyses because empirical evidence (Amarasekare and Coutinho, 2014; Amarasekare, 2015; Johnson et al., 2015; Uszko et al., 2017) suggests this to be the more common form.

## ONLINE APPENDIX C

### Invasibility in a seasonally varying environment

The invasion criterion is given by:

$$\frac{\frac{1}{\tau} \int_0^\tau e_I a_I(T(t)) \frac{d_R(T(t))}{e_R a_R(T(t))} dt}{\frac{1}{\tau} \int_0^\tau d_I(T(t)) dt} > 1. \quad (\text{C.1})$$

where  $e_X$ ,  $a_X(T(t))$  and  $d_X(T(t))$  ( $X = R, I$ ) depict, respectively the resident's and invaders' conversion efficiency and per capita consumption and mortality rates.

We can incorporate the temperature responses of the resident and invader species' resource

consumption and mortality rates (Table 1) to get:

$$\frac{e_I a_{I_{\text{Topt}}}}{d_{I_{\text{TR}}}} \frac{d_{R_{\text{TR}}}}{e_R a_{R_{\text{Topt}}}} \frac{\int_0^\tau e^{-\frac{(T(t)-T_{\text{optaI}})^2}{2s_{aI}^2} + \frac{(T(t)-T_{\text{optaR}})^2}{2s_{aR}^2} + A_{dR} \frac{T(t)-T_{\text{RR}}}{T(t)T_{\text{RR}}}} (1 + e^{A_{LR} \frac{T(t)-T_{\text{LR}}}{T(t)T_{\text{LR}}}}) dt}{\int_0^\tau e^{A_{dI} \frac{T(t)-T_{\text{RI}}}{T(t)T_{\text{RI}}}} (1 + e^{A_{LI} \frac{T(t)-T_{\text{LI}}}{T(t)T_{\text{LI}}}}) dt} > 1, \quad (\text{C.2})$$

Which is Equation (5) in the main text. Note that the subscripts  $R$  and  $I$  denote, respectively, the resident's and invader's trait parameters, and  $(T(t))$  denotes the seasonal temperature regime. The terms outside the integral represent the temperature-independent components of these factors with  $\frac{d_{R_{\text{TR}}}}{e_R a_{R_{\text{Topt}}}}$  depicting the resource availability set by the resident consumer in a constant thermal environment (i.e., its  $R^*$ ), and  $e_I a_{I_{\text{Topt}}}$  and  $d_I$  depicting, respectively, the invader's reproductive and mortality rates. The terms within the integrals determine the temperature-dependent components, with the numerator depicting the resource availability set by the resident, given by the resident's consumption ( $a_R(T(t)) = e^{-\frac{(T(t)-T_{\text{optaR}})^2}{2s_{aR}^2}}$ ) and mortality ( $d_R(T(t)) = e^{A_{dR} \frac{T(t)-T_{\text{RR}}}{T(t)T_{\text{RR}}}} (1 + e^{A_{LR} \frac{T(t)-T_{\text{LR}}}{T(t)T_{\text{LR}}}})$ ) responses, and the invader's resource acquisition ability, given by the temperature response of its consumption rate ( $a_I(T(t)) = e^{-\frac{(T(t)-T_{\text{optaI}})^2}{2s_{aI}^2}}$ ). The denominator depicts the invader's temperature-dependent mortality rate ( $d_I(T(t))$ ).

Now we substitute the characteristics of the seasonal thermal regime ( $(T(t) = M_T - A_T S(t))$ ) and the scaling relationships of the response parameters (e.g.,  $T_{\text{optaR}} = M_T + x$  and  $T_{\text{optaI}} = M_T + x + m$ ; Table C1) into Equation (C.2). Note that the  $M_T$  terms cancel out for the consumption rate responses but not for the mortality responses. By doing some algebra and simplifying, we get the following mechanistic description of the invasion criterion:

$$\frac{f \int_0^\tau e^{-\frac{(A_T S(t)+x+m)^2}{2v^2 s_{aI}^2} + \frac{(A_T S(t)+x)^2}{2s_{aR}^2} - \frac{A_{dR}}{M_T+y} \left( \frac{A_T S(t)+y}{M_T-A_T S(t)} \right)} (1 + e^{-\frac{A_{LR}}{M_T-z} \frac{A_T S(t)-z}{M_T-A_T S(t)}}) dt}{\alpha \delta \int_0^\tau e^{-p_1 \frac{A_{dR}}{M_T+y+g} \left( \frac{A_T S(t)+y+g}{M_T-A_T S(t)} \right)} (1 + e^{-\frac{p_2 A_{LR}}{M_T-z+k} \frac{A_T S(t)-z+k}{M_T-A_T S(t)}}) dt} > 1, \quad (\text{C.3})$$



which is Equation (6) in the main text.

In the numerator of Equation (C.3), the first term inside the integral depicts temperature effects on the invader's resource acquisition ability ( $e^{\frac{-(A_T S(t)+x+m)^2}{2v^2 s_{a_R}^2}}$ ), which is determined by the deviation of its consumption rate optimum from that of the resident ( $m$ ), and the breadth of its consumption response relative to that of the resident ( $v$ ). The second and third terms denote resource availability set by the resident, which results from the interaction between the resident's consumption and mortality responses. The second term denotes the resident's resource acquisition ability, given by its consumption response ( $e^{\frac{(A_T S(t)+x)^2}{2s_{a_R}^2}}$ ), which is determined by the deviation of the resident's thermal optimum from the mean habitat temperature ( $x$ ) and its response breadth ( $s_{a_R}$ ). The third term denotes the resident's mortality response ( $e^{\frac{A_{d_R}}{M_T+y} \left( \frac{A_T S(t)+y}{M_T-A_T S(t)} \right)} \left( 1 + e^{-\frac{A_{d_R}}{M_T-z} \frac{A_T S(t)-z}{M_T-A_T S(t)}} \right)$ ), which depends on the respective rates at which mortality increases with increasing vs. decreasing temperatures ( $A_{d_R}$  and  $A_{d_L}$ ), the deviation of the reference temperature and lower temperature threshold from the mean habitat temperature ( $y$  and  $z$  respectively). In the denominator, the term inside the integral depicts the invader's mortality, which is determined by the respective rates at which its mortality increases with increasing vs. decreasing temperatures ( $p_1 A_{d_R}$  and  $p_2 A_{d_L}$ ) and the deviation of its reference temperature and the lower temperature threshold from those of the resident consumer species ( $g$  and  $k$  respectively).

Table 1: Table C1. Definitions of scaled temperature response parameters

<b>Thermal regime</b>	
$M_T$	Mean habitat temperature
$A_T$	Amplitude of seasonal fluctuations
<b>Resident consumer</b>	
$T_{\text{opt}_{a_R}} = M_T + x$	$x$ = deviation consumption rate optimum from mean habitat temperature
$T_{R_R} = M_T + y$	$y$ = deviation of reference temperature from mean habitat temperature
$T_{L_R} = M_T - z$	$z$ = deviation of low temperature threshold from mean habitat temperature
<b>Invading consumer</b>	
$T_{\text{opt}_{a_I}} = T_{\text{opt}_{a_R}} + m$	$m$ = deviation of invader's consumption rate optimum from that of resident
$s_{a_I} = v s_{a_R}$	$v$ = ratio of invader's and resident's consumption response breadths
$a_{I_{T_{\text{opt}}}} = \alpha a_{R_{T_{\text{opt}}}}$	$\alpha$ = ratio of invader's and resident's maximum consumption rates
$e_I = f e_R$	$f$ = ratio of invader's and resident's conversion efficiencies
$T_{R_I} = T_{R_R} + g$	$g$ = deviation of invader's reference temperature for mortality from that of the resident
$T_{L_I} = T_{L_R} + k$	$k$ = deviation of invader's low temperature threshold from that of the resident
$A_{d_I} = p_1 A_{d_R}$	$p_1$ = ratio of Arrhenius constants above the low temperature threshold
$A_{L_I} = p_2 A_{L_R}$	$p_2$ = ratio of Arrhenius constants below the low temperature threshold
$d_{I_{T_R}} = \delta d_{R_{T_R}}$	$\delta$ = ratio of the invader's baseline mortality rate to that of the resident

## ONLINE APPENDIX D

### Temperature and trait response parameters

Table D1 gives the parameter values used in the model analysis, which are realistic for tropical and temperate insects (Dreyer and Baumgartner, 1996; Morgan et al., 2001; Deutsch et al., 2008; Amarasekare and Savage, 2012; Amarasekare and Johnson, 2017).

Table D1. Temperature response functions and parameter values for tropical and temperate species.

	Tropical	Temperate
Thermal regime	$M_T = 299, A_T = 1.5$	$M_T = 285, A_T = 10$
Resident community: resource species		
$T_{\text{opt}_b}$	300	292
$s_b$	3.0	7.0
$b_{\text{Topt}}$	25	25
$T_{\text{opt}_q}$	300	292
$s_q$	3.0	5.0
$q_{\text{Topt}}$	0.02	0.02
$T_R$	298	292
$T_L$	288-90	273
$A_d$	10000	10000
$A_L$	-15000	-7500
Resident community: consumer species		
$e_R$	1.0	1.0
$s_{a_R}$	3.0	7.0
$a_{R\text{Topt}}$	1.0-5.0	1.0
$d_{R\text{T}_R}$	0.05-0.2	0.1-0.2
$A_{d_R}$	10000	10000
$A_{L_R}$	-15000	-7500
$T_{\text{opt}_{a_R}} = MT + x$	$x = 1$	$x = 7$
$T_{R_R} = M_T + y$	$y = -1$	$y = 7$
$T_{L_R} = M_T - z$	$z = 9$	$z = 12$
Invading consumer		
$s_{a_I} = v s_{a_R}$	$v < 1$	$v > 1$
$a_{I\text{Topt}} = \alpha a_{R\text{Topt}}$	$\alpha = 1.0 - 5.0$	$\alpha = 0.1 - 1.0$
$e_I = f e_R$	$f = 0.1 - 1.0$	$f = 0.1 - 1.0$
$d_{I\text{T}_R} = \delta d_{R\text{T}_R}$	$\delta \leq 1$	$\delta \geq 1$
$A_{d_I} = p_1 A_{d_R}$	$p_1 \leq 2$	$p_1 \leq 2$
$A_{L_I} = p_2 A_{L_R}$	$p_2 \geq 1$	$p_2 \leq 1$
$T_{\text{opt}_{a_I}} = T_{\text{opt}_{a_R}} + m$	$m > 0$	$m < 0$
$T_{R_I} = T_{R_R} + g$	$g > 0$	$g < 0$
$T_{L_I} = T_{L_R} + k$	$k > 0$	$k < 0$

## References

- Amarasekare, A., and J. Johnson. 2017. Evolution of thermal reaction norms in seasonally varying environments. *American Naturalist* 189:E31–E45.
- Amarasekare, P. 2015. Temperature effects on consumer-resource interactions. *Journal of Animal Ecology* 84:665–679.

- Amarasekare, P., and R. Coutinho. 2014. Effects of temperature on intra-specific competition in ectotherms. *American Naturalist* 184:E50–E65.
- Amarasekare, P., and V. Savage. 2012. A mechanistic framework for elucidating the temperature dependence of fitness. *American Naturalist* 179:178–191.
- Armstrong, R., and R. McGehee. 1980. Competitive exclusion. *American Naturalist* 115:151–170.
- Carriere, Y., and G. Boivin. 1997. Evolution of thermal sensitivity of parasitization capacity in egg parasitoids. *Evolution* 51:2028–2032.
- Dannon, E. A., M. Tamo, A. van Huis, and M. Dicke. 2010. Functional response and life history parameters of *Apanteles taragamae*, a larval parasitoid of *Maruca vitrata*. *BioControl* 55:363–378.
- Dell, A., S. Pawar, and V. M. Savage. 2011. Systematic variation in the temperature dependence of physiological and ecological traits. *Proceedings of the National Academy of Sciences* 108:10591–10596.
- Deutsch, C. J., J. Tewksbury, R. B. Huey, K. Sheldon, C. Ghalambor, D. Haak, and P. R. Martin. 2008. Impacts of climate warming on terrestrial ectotherms across latitude. *Proc Natl Acad Sci USA* 105:6668–6672.
- Dreyer, H., and J. Baumgartner. 1996. Temperature influence on cohort parameters and demographic characteristics of the two cowpea coreids *Clavigralla tomentosicollis* and *C. shadabi*. *Entomologia Experimentalis et Applicata* 78:201–213.
- Englund, G., G. Ohlund, C. Hein, and D. S. 2011. Temperature dependence of the functional response. *Ecology Letters* 14:914–921.

- Gillooly, J. F., J. H. Brown, G. B. West, V. M. Savage, and E. L. Charnov. 2001. Effects of size and temperature on metabolic rate. *Science* 293:2248–2251.
- Gillooly, J. F., E. L. Charnov, G. B. West, , V. M. Savage, and J. H. Brown. 2002. Effects of size and temperature on developmental time. *Science* 293:2248–2251.
- Hou, Y., and Z. Weng. 2010. Temperature-dependent development and life table parameters of *Octodonta nipae* (coleoptera: Chrysomelidae). *Environmental Entomology* 39:1676–1684.
- Jandricic, S. E., S. P. Wraight, K. C. Bennett, and J. P. Sanderson. 2010. Developmental times and life table statistics of *Aulacorthum solani* (hemiptera: Aphididae) at six constant temperatures, with recommendations on the application of temperature-dependent development models. *Environmental Entomology* 39:1631–1642.
- Johnson, C., R. Coutinho, E. Berlin, K. Dolphin, J. Heyer, B. Kim, A. Leung, J. Sabellon, and P. Amarasekare. 2015. Effects of temperature and resource variation on insect population dynamics: the bordered plant bug (*Largus californicus*) as a case study. *Functional Ecology* In Press.
- Johnson, F., and I. Lewin. 1946. The growth rate of e. coli in relation to temperature, quinine and coenzyme. *Journal of Cellular and Comparative Physiology* 28:47–75.
- Morgan, D., K. F. A. Walters, and J. N. Aegerter. 2001. Effect of temperature and cultivar on the pea aphid, *Acyrtosiphon pisum* (hemiptera: Aphididae) life history. *Bulletin of Entomological Research* 91:4752.
- Murdoch, W., C. J. Briggs, and N. R. M. 2003. Consumer resource dynamics. Princeton University Press, Princeton New Jersey.

- Ratkowsky, D., J. Olley, and T. Ross. 2005. Unifying temperature effects on the growth rate of bacteria and the stability of globular proteins. *Journal of Theoretical Biology* 233:351–362.
- Savage, V. M., J. F. Gillooly, J. H. Brown, G. B. West, and E. L. Charnov. 2004. Effects of body size and temperature on population growth. *American Naturalist* 163:429–441.
- Schoolfield, R., J. Sharpe, and C. Magnuson. 1981. Non-linear regression of biological temperature-dependent rate models based on absolute reaction-rate theory. *Journal of Theoretical Biology* 88:719–731.
- Sharpe, P., and D. DeMichele. 1977. Reaction kinetics of poikilotherm development. *Journal of Theoretical Biology* 64:649–670.
- Uszko, W., S. Diehl, G. Englund, and P. Amarasekare. 2017. Effects of warming on predator-prey interactions - a resource-based approach and a theoretical synthesis. *Ecology Letters* In Press.

Fig. A1. Dynamics of a pairwise consumer-resource interaction in which the resource experiences self-limitation and the resident consumer exhibits a Type I functional response and the species experience a seasonally varying thermal environment (Equation (A.3), Appendix A). Panels (a) and (b) show, respectively, that the temperature response functions of the resident consumer's attack and mortality rates return to their original values after a period of one year. Panels (c) and (d) show that resource and consumer abundances settle into an annual limit cycle with the same minimum and maximum values attained after a period of one year. The insets in each panel show that all initial conditions converge to the same periodic oscillation, i.e., the oscillations constitute a stationary state for the consumer-resource system. Panel (e) shows the congruence between the instantaneous resource abundance ( $R_{C_R}(t)$ ) in the limit cycle solution of Equation (A.3) and the quantity  $\frac{d_R(T(t))}{e_R a_R(T(t))}$ . Parameter values are as follows. Resource species:  $T_{opt_b} = 292K, s_b = 7.0, b_{T_{opt}} = 25.0, A_d = 10000, A_L = -10000, T_{opt_q} = T_{opt_b}, s_q = 5.0, q_{T_{opt}} = 0.02$ ; consumer species:  $T_{opt_{a_R}} = 292K, s_{a_R} = 7.0, a_{R_{T_{opt}}} = 1.0, e_R = 1.0, A_{d_R} = 10000, A_{L_R} = -10000, T_{R_R} = 292K, T_{L_R} = 273K, d_{R_{T_R}} = 0.2, M_T = 285K, A_T = 10$ .

Fig. A2. Comparison between the instantaneous resource abundance  $R_{C_R}(t)$  in the limit cycle solution of Equation (A.3) and the quantity  $\frac{d_R(T(t))}{e_R a_R(T(t))}$  ( $t = 0, \dots, 365$ ) over key parameter values of the temperature response functions. In all panels, the black circles depict the statistical correlation between the two time series. Panel (a) depicts the correlation between the two time series as a function of maximum resource birth rate ( $b_{T_{opt}}$ ) and response breadth ( $s_b$ ), panel (b), as a function of maximum resource birth rate ( $b_{T_{opt}}$ ) and the intrinsic mortality rate of the resident consumer ( $d_{R_{T_R}}$ ), panel (c), the ratios of temperature sensitivity of mortality below and above the low temperature threshold ( $\frac{A_{d_R}}{A_d}$  and  $\frac{A_{L_R}}{A_L}$ ) and panel (d), maximum consumer attack rate ( $a_{R_{T_{opt}}}$ ) and the ratio of the attack response breadth to that of resource birth rate ( $\frac{s_{a_R}}{s_b}$ ). Note that the correlation between the limit cycle solution and  $\frac{d_R(T(t))}{e_R a_R(T(t))}$  approaches unity when

the consumer is well adapted to the resource, i.e., maximum resource birth rate and/or response breadth are high enough to allow the consumer to persist on the resource, the consumer's intrinsic mortality rate is high enough to prevent resource overexploitation, its temperature sensitivity of mortality above the low temperature threshold does not exceed that of the resource, and its maximum attack rate and/or response breadth are high enough for it to subsist on the resource without overexploiting it. Parameter combinations for which data are missing are those that do not allow resource-consumer coexistence. Common parameter values are as follows. Resource species:  $T_{opt_b} = 292K$ ,  $A_d = 10000$ ,  $A_L = -25000$ ,  $T_{opt_q} = T_{opt_b}$ ,  $s_q = s_b$ ,  $q_{T_{opt}} = 0.02$ ; consumer species:  $T_{opt_{a_R}} = 292K$ ,  $s_{a_R} = 7.0$ ,  $a_{R_{T_{opt}}} = 1.0$ ,  $e_1 = 1.0$ ,  $A_{d_R} = 10000$ ,  $A_{L_R} = -25000$ ,  $T_{R_R} = 292K$ ,  $T_{L_R} = 273K$ ,  $M_T = 285K$ ,  $A_T = 10$ .



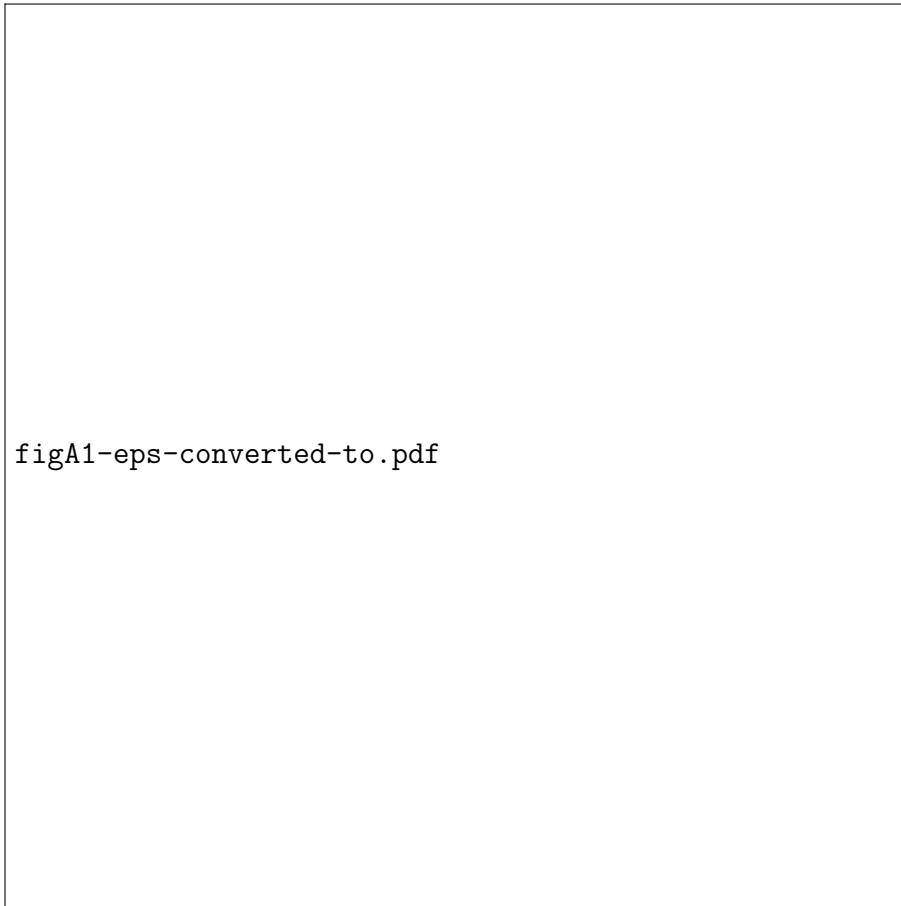


Figure A1



Figure A2