

```

(*)
This program proves that the self-duality implies MA property;
u=U[x1,x2,x3,x4];

ui=D[u,xi];

uij=D[u,xi,xj];

u11=f[u12,u13,u14,u22,u23,u24,u33,u34,u44,u1,u2,u3,u4,u,x1,x2,x3,x4];

G - metric with upper indecies;
g - metric with lower indecies;

Sub - factorisation list modulo u11=
  f[u12,u13,u14,u22,u23,u24,u33,u34,u44,u1,u2,u3,u4,u,x1,x2,x3,x4];

pt - point;

We compute the symbol of Weyl tensor W
  (coefficient of W at fourth derivatives of u),
as well as symbols of Wplus and Wminus. We compute systems of equations
  equivalent to conditions that symbols of W, Wplus, Wminus vanish;

glist - list of elements of  $\Gamma[i,j,k]$  - connection;

Rlist - list of elements of the Riemann tensor; In computing Rlist
  we drop terms depending on derivatives of u of orders less than 4;

S - scalar curvature;

Wlist - list of elements of the Weyl tensor;

Wlist3 -
  list of elements of the Weyl tensor depending only on 4th derivatives of u;

sysWeyl - system of conditions equivalent to  $W=0$ ;

WS -  $W^*$ ;

Wpluslist, Wminuslist - lists of elements of Wplus, Wminus;

sysplus, sysminus - systems of conditions equivalent to  $Wplus=0$ ,  $Wminus=0$ ;

Lid - list of Monge -
  Ampere equations ( $a_1$ - $a_{25}$  - parameters, MAE hold when  $a_i=0$ ,  $i=1,\dots,25$ );

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We show that either of systems $\text{sysplus} = 0$ and $\text{sysminus} = 0$ imply $a_i = 0$, $i = 1, \dots, 25$.

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u = U[x1, x2, x3, x4];
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u1 = D[u, x1];
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u2 = D[u, x2];
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u3 = D[u, x3];
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u4 = D[u, x4];
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u11 = D[u1, x1];
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u12 = D[u1, x2];
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u13 = D[u1, x3];
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u14 = D[u1, x4];
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u22 = D[u2, x2];
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u23 = D[u2, x3];
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u24 = D[u2, x4];
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u33 = D[u3, x3];
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```
u34 = D[u3, x4];
```

```
u44 = D[u4, x4];
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f = F[u12, u13, u14, u22, u23, u24, u33, u34, u44, u1, u2, u3, u4, u, x1, x2, x3, x4];
```

```
G = {{-1, 1/2 D[f, u12], 1/2 D[f, u13], 1/2 D[f, u14]},
      {1/2 D[f, u12], D[f, u22], 1/2 D[f, u23], 1/2 D[f, u24]},
      {1/2 D[f, u13], 1/2 D[f, u23], D[f, u33], 1/2 D[f, u34]},
      {1/2 D[f, u14], 1/2 D[f, u24], 1/2 D[f, u34], D[f, u44]}};
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together[a_, b_] :=
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Block[{res = 0, lcm = 1}, lcm = PolynomialLCM[Denominator[a], Denominator[b]];
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res = (Factor[lcm/Denominator[a]] Numerator[a] +
       Factor[lcm/Denominator[b]] Numerator[b])/lcm];
```

```
g = Inverse[G] // Factor;
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var = {x1, x2, x3, x4};
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Sub2 = {u11 → f};
sys = {};
elem = Sub2[[1]][[1]] - Sub2[[1]][[2]];
Do[AppendTo[sys, Numerator[Factor[D[elem, var[[i]]] /. Sub2]]], {i, 1, 4}];
vars3 = {};
Do[AppendTo[vars3, D[u11, var[[i]]]], {i, 1, 4}];
Sub3 = Solve[sys == 0, vars3][[1]] // Factor;
vars4 = {};
Do[Do[AppendTo[vars4, D[u11, var[[i]], var[[j]]]], {j, i, 4}], {i, 1, 4}];
sys = {};
Do[elem = Sub3[[i]][[1]] - Sub3[[i]][[2]];
  Do[AppendTo[sys, Numerator[Factor[D[elem, var[[j]]] /. Sub2 /. Sub3]]],
    {j, 1, 4}], {i, 1, Length[Sub3]};
Sub4 = Solve[sys == 0, vars4][[1]] // Factor;
Sub = Flatten[{Sub2, Sub3, Sub4}];

der[exp_, n_] :=
  ((D[Numerator[exp], var[[n]]] Denominator[exp] - D[Denominator[exp], var[[n]]]
    Numerator[exp]) /. Sub) / Denominator[exp]^2;

list1 = {};
var1 = {u12, u13, u14, u22, u23, u24, u33, u34, u44};
var1s = {"12", "13", "14", "22", "23", "24", "33", "34", "44"};
Do[el = D[f, var1[[i]]] → ToExpression["f" <> var1s[[i]]];
  AppendTo[list1, el], {i, 1, Length[var1]}];

list2 = {};
var1 = {u12, u13, u14, u22, u23, u24, u33, u34, u44};
var1s = {"12", "13", "14", "22", "23", "24", "33", "34", "44"};
Do[
  Do[el = D[f, var1[[i]], var1[[j]]] → ToExpression["f" <> var1s[[i]] <> var1s[[j]]];
    AppendTo[list2, el], {j, i, Length[var1]}], {i, 1, Length[var1]}];

SS = Flatten[{list1, list2}];

pt = {f12 → 0, f13 → 0, f14 → 1, f22 → 0, f23 → 1, f24 → 0, f33 → 0, f34 → 0, f44 → 0};

Det[g /. SS /. pt]

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Rm[k_, i_, j_] := Block[{res = 0}, Do[
  res = together[res, 1/2 G[[k]][[l]] der[g[[l]][[j]], i]];
  res = together[res, 1/2 G[[k]][[l]] der[g[[i]][[l]], j]];
  res = together[res, -1/2 G[[k]][[l]] der[g[[i]][[j]], l], {l, 1, 4}];
Factor[res /. SS /. pt]];

```

```

glist = {};
Do[
  Do[Do[AppendTo[glist, R[k, i, j] → Rm[k, i, j]], {j, 1, 4}], {i, 1, 4}], {k, 1, 4}];

```

```

Rm[i_, q_, k_, l_] :=
  Block[{res = 0}, res = together[res, der[(R[i, q, k] /. glist), l]];
  res = together[res, -der[(R[i, q, l] /. glist), k]];
  (*Do[res=res+(R[i,p,l]/.glist)(R[p,q,k]/.glist)-(R[i,p,k]/.glist)
    (R[p,q,l]/.glist),{p,1,4}];*) res = Factor[(res /. SS /. pt)];
  res];

```

```

ders3 = {};
Do[Do[Do[el = D[U[x1, x2, x3, x4], var[[i]], var[[j]], var[[k]]] → 0;
  AppendTo[ders3, el], {k, j, 4}], {j, i, 4}], {i, 1, 4}]

```

```

Rlist = {};
Do[Do[Do[Do[js = Rm[i, j, k, l] /. ders3;
  el = R[i, j, k, l] → js;
  AppendTo[Rlist, el];
  el = R[i, j, l, k] → -js;
  AppendTo[Rlist, el], {l, k+1, 4}];
  el = R[i, j, k, k] → 0;
  AppendTo[Rlist, el], {k, 1, 4}], {j, 1, 4}], {i, 1, 4}];

```

```

RRm[q_, l_] := Block[{}, el = 0;
  Do[el = together[el, (R[i, q, i, l] /. Rlist)], {i, 1, 4}];
  el]

```

```

RRlist = {};
Do[Do[AppendTo[RRlist, RR[q, l] → RRm[q, l]], {l, 1, 4}], {q, 1, 4}]

```

```

(*S*)
S = 0;
Do[Do[S = together[S, (G[[l]][[q]] /. SS /. pt) (RR[q, l] /. RRlist /. pt)],
  {q, 1, 4}], {l, 1, 4}];

R1[i_, j_, k_, l_] := Block[{res = 0}, Do[res =
  together[res, (g[[i]][[s]] /. SS /. pt) (R[s, j, k, l] /. Rlist)], {s, 1, 4}];
res];

w[i_, j_] := together[1/2 (RR[i, j] /. RRlist), -S/12 (g[[i]][[j]] /. SS /. pt)];

Wm[i_, j_, k_, l_] := Block[{res = 0}, res = together[res, (R1[i, j, k, l])];
  res = together[res, -w[i, k] (g[[j]][[l]] /. SS /. pt)];
  res = together[res, -w[j, l] (g[[i]][[k]] /. SS /. pt)];
  res = together[res, +w[j, k] (g[[i]][[l]] /. SS /. pt)];
  res = together[res, +w[i, l] (g[[j]][[k]] /. SS /. pt)];
  res];

Wlist = {};
Do[Do[Do[el = W[i, j, k, k] → 0;
  AppendTo[Wlist, el];
  el = W[k, k, i, j] → 0;
  AppendTo[Wlist, el], {k, 1, 4}], {j, 1, 4}], {i, 1, 4}]

```

```

Do[Do[Do[Do[js = W[i, j, k, l];
  el = W[i, j, k, l] → js;
  AppendTo[Wlist, el];
  el = W[i, j, l, k] → -js;
  AppendTo[Wlist, el];
  el = W[j, i, k, l] → -js;
  AppendTo[Wlist, el];
  el = W[j, i, l, k] → js;
  AppendTo[Wlist, el];
  el = W[k, l, i, j] → js;
  AppendTo[Wlist, el];
  el = W[k, l, j, i] → -js;
  AppendTo[Wlist, el];
  el = W[l, k, i, j] → -js;
  AppendTo[Wlist, el];
  el = W[l, k, j, i] → js;
  AppendTo[Wlist, el], {l, j + (k + 1 - j) Sign[k - i], 4}], {k, i, 4}],
{j, i + 1, 4}], {i, 1, 4}];
Wlist = Union[Wlist];

```

```
Length[Wlist]
```

```
256
```

```

Wlist1 = {};
Do[Do[Do[Do[js = Wm[i, j, k, l];
  el = W[i, j, k, l] → js;
  AppendTo[Wlist1, el], {l, j + (k + 1 - j) Sign[k - i], 4}], {k, i, 4}],
{j, i + 1, 4}], {i, 1, 4}];

```

```
Length[Wlist1]
```

```
21
```

```
Wlist2 = Wlist1;
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ders4 = {};
Do[Do[Do[Do[el = D[U[x1, x2, x3, x4], var[[i]], var[[j]], var[[k]], var[[l]]];
  AppendTo[ders4, el], {l, k, 4}], {k, j, 4}], {j, i, 4}], {i, 1, 4}];
ders4sub = {};
Do[el = ders4[[i]] → λ ders4[[i]];
  AppendTo[ders4sub, el], {i, 1, Length[ders4]}];

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Wlist3 = {};
Do[el = Wlist1[[i]][[1]] → Coefficient[(Wlist1[[i]][[2]] /. ders4sub), λ, 1];
  AppendTo[Wlist3, el], {i, 1, Length[Wlist1]};

sysWeyl = {};
Do[Do[AppendTo[sysWeyl, Coefficient[Wlist3[[i]][[2]], ders4[[j]]]],
  {j, 1, Length[ders4]}], {i, 1, Length[Wlist3]}]

SolsWeyl = ToRules[Reduce[sysWeyl == 0]]
{f3333 → 0, f2222 → 0, f3434 → 0, f3344 → 0, f1433 → 0, f1334 → f2333,
 f1313 → -2 f2333, f2224 → 0, f1222 → 0, f3334 → 0, f1333 → 0, f2444 → 0,
 f1424 → f2324, f1322 → -f2234 + f2324, f1244 → f2234, f1223 → f2234 - f2324,
 f1214 → -f2234 - f2324, f3444 → 0, f1434 → f2334, f1344 → f2433,
 f1323 → -f2334 + f2433, f1314 → -f2334 - f2433, f1233 → f2334 - f2433, f4444 → 0,
 f1444 → f2344 + f2434, f1423 → -f2233 +  $\frac{f2323}{2}$  - 2 f2344, f1414 → -2 f2344 - 2 f2434,
 f1324 → f2233 + f2344, f1234 → f2233 + f2344, f1213 → -f2233 -  $\frac{f2323}{2}$ ,
 f2424 → 0, f2244 → 0, f1422 → 0, f1224 → f2223, f1212 → -2 f2223}

Length[SolsWeyl]
35

Clear[ε]

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```

el1 = {};
Do[Do[Do[AppendTo[el1,  $\epsilon[i1, i2, i3, i4] \rightarrow 0$ ], {i4, 1, 4}], {i3, 1, 4}],
  {i2, 1, 4}], {i1, 1, 4}]

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```

el1 = {};
AppendTo[el1, { $\epsilon[1, 2, 3, 4] \rightarrow 1$ ,
   $\epsilon[1, 2, 4, 3] \rightarrow -1$ ,
   $\epsilon[1, 3, 2, 4] \rightarrow -1$ ,
   $\epsilon[1, 3, 4, 2] \rightarrow 1$ ,
   $\epsilon[1, 4, 2, 3] \rightarrow 1$ ,
   $\epsilon[1, 4, 3, 2] \rightarrow -1$ ,
   $\epsilon[2, 1, 3, 4] \rightarrow -1$ ,
   $\epsilon[2, 1, 4, 3] \rightarrow 1$ ,
   $\epsilon[2, 3, 1, 4] \rightarrow 1$ ,
   $\epsilon[2, 3, 4, 1] \rightarrow -1$ ,
   $\epsilon[2, 4, 1, 3] \rightarrow -1$ ,
   $\epsilon[2, 4, 3, 1] \rightarrow 1$ ,
   $\epsilon[3, 1, 2, 4] \rightarrow 1$ ,
   $\epsilon[3, 1, 4, 2] \rightarrow -1$ ,
   $\epsilon[3, 2, 1, 4] \rightarrow -1$ ,
   $\epsilon[3, 2, 4, 1] \rightarrow 1$ ,
   $\epsilon[3, 4, 1, 2] \rightarrow 1$ ,
   $\epsilon[3, 4, 2, 1] \rightarrow -1$ ,
   $\epsilon[4, 1, 2, 3] \rightarrow -1$ ,
   $\epsilon[4, 1, 3, 2] \rightarrow 1$ ,
   $\epsilon[4, 2, 1, 3] \rightarrow 1$ ,
   $\epsilon[4, 2, 3, 1] \rightarrow -1$ ,
   $\epsilon[4, 3, 1, 2] \rightarrow -1$ ,
   $\epsilon[4, 3, 2, 1] \rightarrow 1$ }}];
el1 = Flatten[el1];

```

```
Det[G /. SS /. pt]
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$$\frac{1}{16}$$

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WW[i_, j_, k_, l_] := Block[{res, a, b}, res = 0;
  Do[Do[res = together[res, (G[[i]][[a]] /. SS /. pt) (G[[j]][[b]] /. SS /. pt)
    (W[a, b, k, l] /. Wlist /. Wlist3)], {b, 1, 4}], {a, 1, 4}];
  Factor[res]];

Gsqrtr = 1 / 4;

Vol[i_, j_, k_, l_] := (e[i, j, k, l] /. elist1 /. elist) / Gsqrtr;

WS[i_, j_, k_, l_] := Block[{res = 0, a, b}, Do[Do[
  res = together[res, 1/2 Vol[i, j, a, b] WW[a, b, k, l]], {b, 1, 4}], {a, 1, 4}];
  res];

Wplus[i_, j_, k_, l_] :=
  1/2 (together[(W[i, j, k, l] /. Wlist /. Wlist3), WS[i, j, k, l]]);
Wminus[i_, j_, k_, l_] :=
  1/2 (together[(W[i, j, k, l] /. Wlist /. Wlist3), -WS[i, j, k, l]]);

Wpluslist = {};
Do[
  Do[Do[Do[AppendTo[Wpluslist, Factor[Wplus[i, j, k, l]]], {l, 1, 4}], {k, 1, 4}],
    {j, 1, 4}], {i, 1, 4}];

Wpluslist = Complement[Wpluslist, {0}];

Wminuslist = {};
Do[
  Do[Do[Do[AppendTo[Wminuslist, Factor[Wminus[i, j, k, l]]], {l, 1, 4}], {k, 1, 4}],
    {j, 1, 4}], {i, 1, 4}];
Wminuslist = Complement[Wminuslist, {0}];

sysplus = {};
Do[Do[var = ders4[[j]];
  varc = Complement[ders4, {var}];
  el = Numerator[Wpluslist[[i]]] /. var → 1;
  Do[el = el /. varc[[k]] → 0, {k, 1, Length[varc]}];
  el = Factor[el];
  AppendTo[sysplus, el]
  , {j, 1, Length[ders4]}], {i, 1, Length[Wpluslist]}];

```

Length[sysplus]

350

ToRules[Reduce[sysplus == 0]]

$$\left\{ \begin{aligned} &f_{2222} \rightarrow 0, f_{2224} \rightarrow 0, f_{1222} \rightarrow 0, f_{3334} \rightarrow 0, f_{1333} \rightarrow 0, f_{3444} \rightarrow 0, f_{1434} \rightarrow f_{2334}, \\ &f_{1344} \rightarrow f_{2433}, f_{1314} \rightarrow -f_{2334} - f_{2433}, f_{1233} \rightarrow -f_{1323}, f_{3333} \rightarrow 0, \\ &f_{3434} \rightarrow 0, f_{3344} \rightarrow 0, f_{1334} \rightarrow -f_{1433} + f_{2333}, f_{1313} \rightarrow -2 f_{2333}, f_{2444} \rightarrow 0, \\ &f_{1322} \rightarrow f_{1424} - f_{2234}, f_{1244} \rightarrow -f_{1424} + f_{2234} + f_{2324}, f_{1223} \rightarrow -f_{1424} + f_{2234}, \\ &f_{1214} \rightarrow -f_{2234} - f_{2324}, f_{4444} \rightarrow 0, f_{1444} \rightarrow f_{2344} + f_{2434}, f_{1414} \rightarrow -2 f_{2344} - 2 f_{2434}, \\ &f_{1324} \rightarrow f_{2233} + f_{2344}, f_{1234} \rightarrow -f_{1423} + \frac{f_{2323}}{2} - f_{2344}, f_{1213} \rightarrow -f_{2233} - \frac{f_{2323}}{2}, \\ &f_{2244} \rightarrow -\frac{f_{2424}}{2}, f_{1422} \rightarrow \frac{f_{2424}}{2}, f_{1224} \rightarrow f_{2223} - \frac{f_{2424}}{2}, f_{1212} \rightarrow -2 f_{2223} \end{aligned} \right\}$$

sysminus = {};

Do[Do[var = ders4[[j]];

varc = Complement[ders4, {var}];

el = Numerator[Wminuslist[[i]]] /. var → 1;

Do[el = el /. varc[[k]] → 0, {k, 1, Length[varc]}];

el = Factor[el];

AppendTo[sysminus, el]

, {j, 1, Length[ders4]}], {i, 1, Length[Wminuslist]}]

ToRules[Reduce[sysminus == 0]]

$$\left\{ \begin{aligned} &f_{2222} \rightarrow 0, f_{3333} \rightarrow 0, f_{2224} \rightarrow 0, f_{1222} \rightarrow 0, f_{3334} \rightarrow 0, f_{1333} \rightarrow 0, f_{3344} \rightarrow -\frac{f_{3434}}{2}, \\ &f_{1433} \rightarrow \frac{f_{3434}}{2}, f_{1334} \rightarrow f_{2333} - \frac{f_{3434}}{2}, f_{1313} \rightarrow -2 f_{2333}, f_{3444} \rightarrow 0, \\ &f_{1344} \rightarrow -f_{1434} + f_{2334} + f_{2433}, f_{1323} \rightarrow -f_{1434} + f_{2433}, f_{1314} \rightarrow -f_{2334} - f_{2433}, \\ &f_{1233} \rightarrow f_{1434} - f_{2433}, f_{2444} \rightarrow 0, f_{1424} \rightarrow f_{2324}, f_{1244} \rightarrow f_{2234}, f_{1223} \rightarrow -f_{1322}, \\ &f_{1214} \rightarrow -f_{2234} - f_{2324}, f_{4444} \rightarrow 0, f_{1444} \rightarrow f_{2344} + f_{2434}, f_{1414} \rightarrow -2 f_{2344} - 2 f_{2434}, \\ &f_{1324} \rightarrow -f_{1423} + \frac{f_{2323}}{2} - f_{2344}, f_{1234} \rightarrow f_{2233} + f_{2344}, f_{1213} \rightarrow -f_{2233} - \frac{f_{2323}}{2}, \\ &f_{2424} \rightarrow 0, f_{2244} \rightarrow 0, f_{1224} \rightarrow -f_{1422} + f_{2223}, f_{1212} \rightarrow -2 f_{2223} \end{aligned} \right\}$$

ToRules[Reduce[sysplus == 0]]

$$\{ \begin{aligned} & f_{2222} \rightarrow 0, f_{2224} \rightarrow 0, f_{1222} \rightarrow 0, f_{3334} \rightarrow 0, f_{1333} \rightarrow 0, f_{3444} \rightarrow 0, f_{1434} \rightarrow f_{2334}, \\ & f_{1344} \rightarrow f_{2433}, f_{1314} \rightarrow -f_{2334} - f_{2433}, f_{1233} \rightarrow -f_{1323}, f_{3333} \rightarrow 0, \\ & f_{3434} \rightarrow 0, f_{3344} \rightarrow 0, f_{1334} \rightarrow -f_{1433} + f_{2333}, f_{1313} \rightarrow -2 f_{2333}, f_{2444} \rightarrow 0, \\ & f_{1322} \rightarrow f_{1424} - f_{2234}, f_{1244} \rightarrow -f_{1424} + f_{2234} + f_{2324}, f_{1223} \rightarrow -f_{1424} + f_{2234}, \\ & f_{1214} \rightarrow -f_{2234} - f_{2324}, f_{4444} \rightarrow 0, f_{1444} \rightarrow f_{2344} + f_{2434}, f_{1414} \rightarrow -2 f_{2344} - 2 f_{2434}, \\ & f_{1324} \rightarrow f_{2233} + f_{2344}, f_{1234} \rightarrow -f_{1423} + \frac{f_{2323}}{2} - f_{2344}, f_{1213} \rightarrow -f_{2233} - \frac{f_{2323}}{2}, \\ & f_{2244} \rightarrow -\frac{f_{2424}}{2}, f_{1422} \rightarrow \frac{f_{2424}}{2}, f_{1224} \rightarrow f_{2223} - \frac{f_{2424}}{2}, f_{1212} \rightarrow -2 f_{2223} \} \end{aligned}$$

Length[ToRules[Reduce[sysplus == 0 && sysminus == 0]]]

35

$$\begin{aligned} \text{Lid} = \{ & f_{2222} \rightarrow a_1 - f_{1212} f_{22}, f_{3333} \rightarrow a_2 - f_{1313} f_{33}, \\ & f_{4444} \rightarrow a_3 - f_{1414} f_{44}, f_{1222} \rightarrow \frac{1}{2} (a_4 - f_{12} f_{1212}), f_{1333} \rightarrow \frac{1}{2} (a_5 - f_{13} f_{1313}), \\ & f_{1444} \rightarrow \frac{1}{2} (a_6 - f_{14} f_{1414}), f_{1322} \rightarrow \frac{1}{2} (a_7 - 2 f_{12} f_{1213} - 2 f_{1223} - f_{1212} f_{13}), \\ & f_{1323} \rightarrow \frac{1}{2} (a_8 - 2 f_{1233} - 2 f_{1213} f_{13} - f_{12} f_{1313}), \\ & f_{1422} \rightarrow \frac{1}{2} (a_9 - 2 f_{12} f_{1214} - 2 f_{1224} - f_{1212} f_{14}), \\ & f_{1424} \rightarrow \frac{1}{2} (a_{10} - 2 f_{1244} - 2 f_{1214} f_{14} - f_{12} f_{1414}), \\ & f_{1433} \rightarrow \frac{1}{2} (a_{11} - 2 f_{13} f_{1314} - 2 f_{1334} - f_{1313} f_{14}), \\ & f_{1434} \rightarrow \frac{1}{2} (a_{12} - 2 f_{1344} - 2 f_{1314} f_{14} - f_{13} f_{1414}), \\ & f_{2223} \rightarrow \frac{1}{2} (a_{13} - 2 f_{1213} f_{22} - f_{1212} f_{23}), f_{2333} \rightarrow \frac{1}{2} (a_{14} - f_{1313} f_{23} - 2 f_{1213} f_{33}), \\ & f_{2224} \rightarrow \frac{1}{2} (a_{15} - 2 f_{1214} f_{22} - f_{1212} f_{24}), f_{2444} \rightarrow \frac{1}{2} (a_{16} - f_{1414} f_{24} - 2 f_{1214} f_{44}), \\ & f_{3334} \rightarrow \frac{1}{2} (a_{17} - 2 f_{1314} f_{33} - f_{1313} f_{34}), f_{3444} \rightarrow \frac{1}{2} (a_{18} - f_{1414} f_{34} - 2 f_{1314} f_{44}), \\ & f_{2323} \rightarrow a_{19} - f_{1313} f_{22} - 2 f_{2233} - 2 f_{1213} f_{23} - f_{1212} f_{33}, \\ & f_{2424} \rightarrow a_{20} - f_{1414} f_{22} - 2 f_{2244} - 2 f_{1214} f_{24} - f_{1212} f_{44}, \\ & f_{3434} \rightarrow a_{21} - f_{1414} f_{33} - 2 f_{3344} - 2 f_{1314} f_{34} - f_{1313} f_{44}, \\ & f_{1423} \rightarrow a_{22} - f_{1234} - f_{1214} f_{13} - f_{12} f_{1314} - f_{1324} - f_{1213} f_{14}, \\ & f_{2234} \rightarrow \frac{1}{2} (a_{23} - 2 f_{1314} f_{22} - 2 f_{1214} f_{23} - 2 f_{2324} - 2 f_{1213} f_{24} - f_{1212} f_{34}), \\ & f_{2334} \rightarrow \frac{1}{2} (a_{24} - 2 f_{1314} f_{23} - f_{1313} f_{24} - 2 f_{2433} - 2 f_{1214} f_{33} - 2 f_{1213} f_{34}), \\ & f_{2344} \rightarrow \frac{1}{2} (a_{25} - f_{1414} f_{23} - 2 f_{1314} f_{24} - 2 f_{2434} - 2 f_{1214} f_{34} - 2 f_{1213} f_{44}) \} /. \text{pt}; \end{aligned}$$

```
(*sysplus==0 ==> ai=0, i=1,...,25*)
```

```
ToRules[Reduce[(sysplus /. Lid) == 0]]
```

```
{a4 → 0, a15 → 0, f1212 → -2 f1224 + 2 f2244, a9 → 0, a20 → 0,
a13 → 0, a2 → 0, a5 → 0, a17 → 0, f1344 → f2433, a8 → 0, a24 → 0,
a18 → 0, a12 → 0, f1324 → - $\frac{f1414}{2}$  + f2233 - f2434, a6 → 0, a3 → 0,
a25 → 0, a22 → 0, a19 → 0, f1223 → f1244 - f2324, a7 → 0, a23 → 0,
a16 → 0, a10 → 0, f3344 → 0, a21 → 0, a14 → 0, a11 → 0, a1 → 0}
```

```
(*sysminus==0 ==> ai=0, i=1,...,25*)
```

```
ToRules[Reduce[(sysminus /. Lid) == 0]]
```

```
{a4 → 0, a15 → 0, f2244 → 0, a9 → 0, a20 → 0, a13 → 0, a2 → 0, a5 → 0,
a17 → 0, f1233 → -f1314 - f1344 - f2433, a8 → 0, a24 → 0, a18 → 0,
a12 → 0, f1234 → - $\frac{f1414}{2}$  + f2233 - f2434, a6 → 0, a3 → 0, a25 → 0,
a22 → 0, a19 → 0, f1214 → -f1244 - f2324, a7 → 0, a23 → 0, a16 → 0,
a10 → 0, f1313 → -2 f1334 + 2 f3344, a21 → 0, a14 → 0, a11 → 0, a1 → 0}
```