

THE EFFECT OF NETWORK TOPOLOGY ON OPTIMAL EXPLORATION STRATEGIES AND THE EVOLUTION OF COOPERATION IN A MOBILE POPULATION: SUPPLEMENTARY INFORMATION

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1. SCENARIO 1: INTERACTIVE STRATEGY MUTATIONS ARE RARE

In this scenario, we vary the population size, exploration time, and the reward-to-cost ratio. We present the results for each of the three network structures (complete, circle, and star graphs), and within each structure we consider the effects of the population size, exploration time, and the reward-to-cost ratio in turn, while all other parameters assume their base values.

1.1. Complete graph. In figure 1 we see the optimal staying propensities and fixation probabilities for the complete graph over the full range of population sizes that we considered (i.e., 10–50). The staying propensities behave in essentially the same way for all population sizes. Resident defectors use (as in all cases) the maximum value of 0.99, with mutant cooperators increasing from a high level to the maximum as the movement cost increases. Resident cooperators vary between a staying propensity of 0.2 for low cost to one of 0.6 for a high cost; mutant defectors increase from the minimum value of 0.01 to the maximum of 0.99, with intermediate values over a small range of costs only. In all cases selection favours cooperators for movement costs up to 0.2. Selection favours defectors only in sufficiently small populations and for sufficiently large movement costs.

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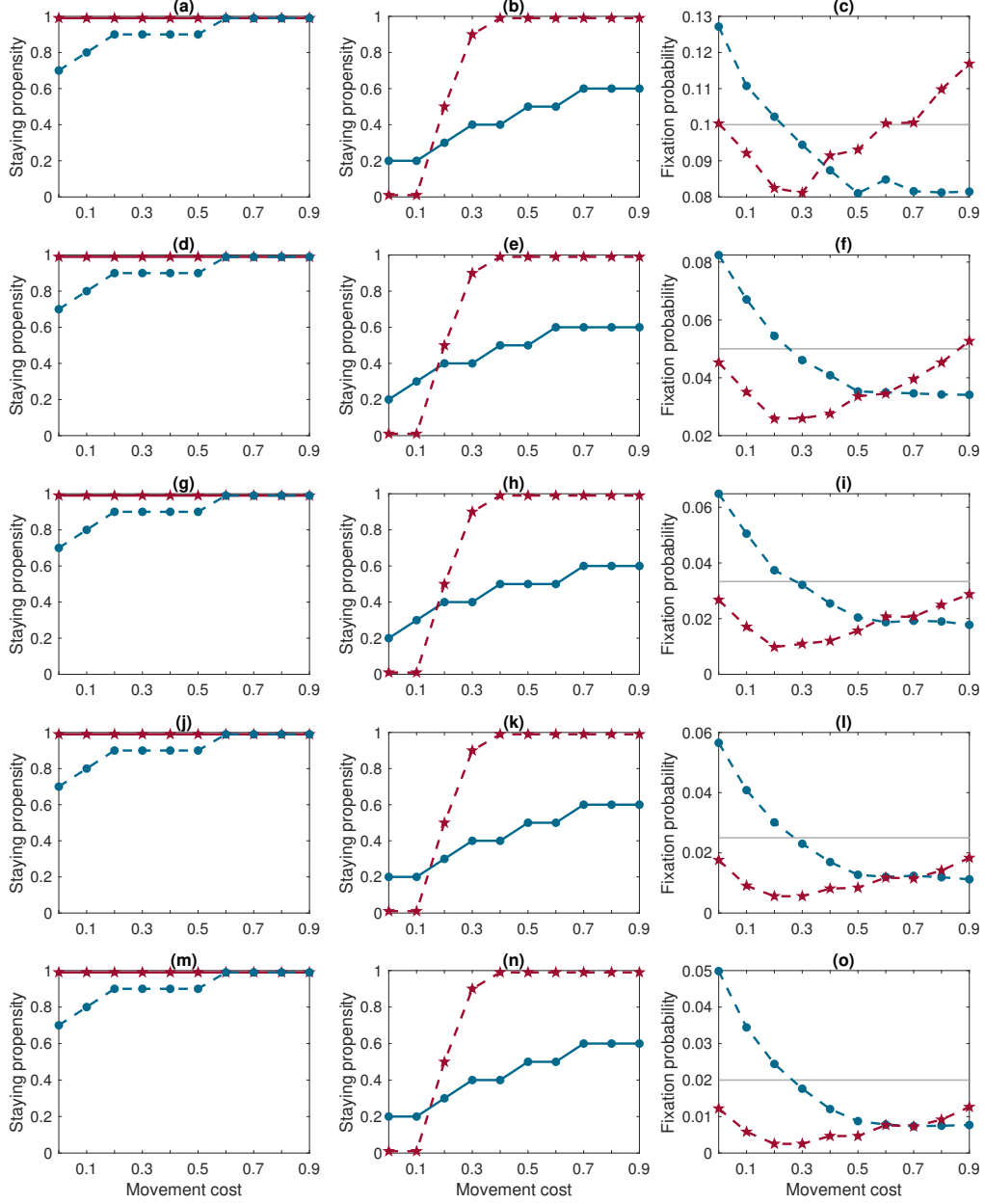


Figure 1. Varying the population size for the complete graph: (a)–(c) population size 10; (d)–(f) population size 20; (g)–(i) population size 30; (j)–(l) population size 40; (m)–(o) population size 50. The left-column panels show optimal staying propensities for resident defectors and mutant cooperators, the middle-column panels show optimal staying propensities for resident cooperators and mutant defectors, and the right-column panels show the fixation probabilities of best mutant cooperators and best mutant defectors in the resident defector and cooperator populations, respectively. Legend: defectors—red with star markers for data points, cooperators—blue with round markers for data points; residents—solid line, mutants—dashed line.

Changing exploration time has little effect on the staying propensities. It is clear that longer exploration times are favourable to cooperators, who have more time to find and benefit from other cooperators. In particular, shortening the exploration time as in figure 2 means selection favours defectors for sufficiently high movement cost even in large populations, whilst lengthening the exploration time as in figure 3 results in selection favouring cooperators up to a higher movement cost threshold.

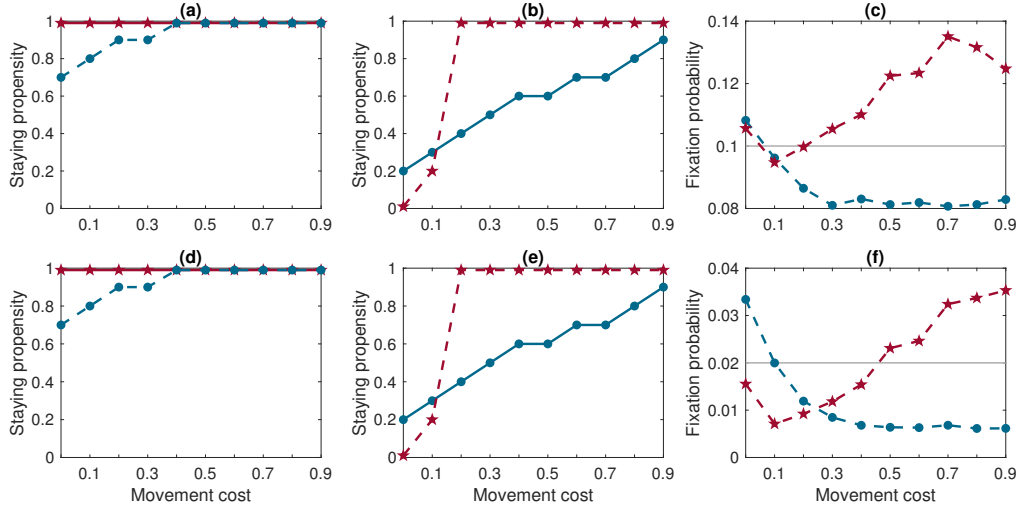


Figure 2. Short exploration time $T = 5$ for the complete graph: (a)–(c) population size 10; (d)–(f) population size 50. For legend see the figure 1 caption.

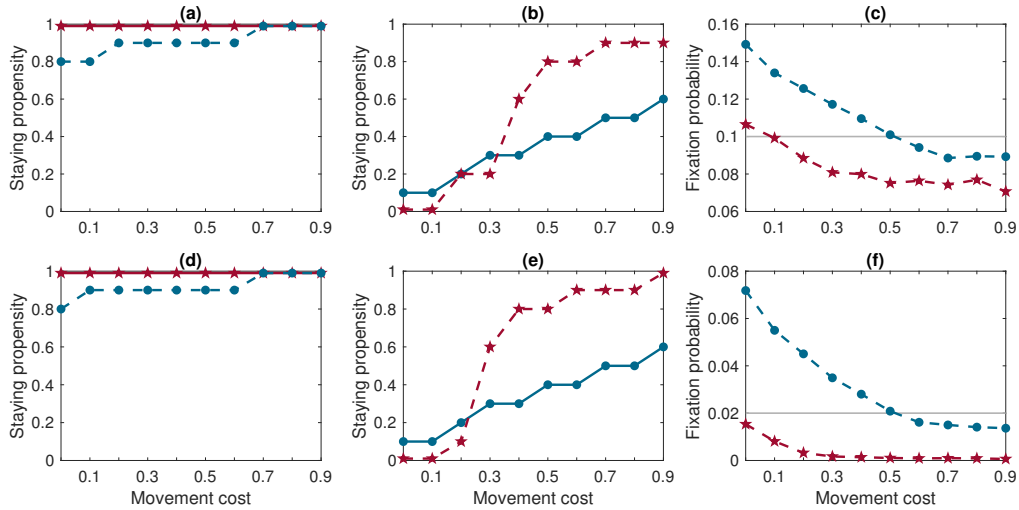


Figure 3. Long exploration time $T = 25$ for the complete graph: (a)–(c) population size 10; (d)–(f) population size 50. For legend see the figure 1 caption.

In figures 4 (population size 10) and 5 (population size 50) we consider varying the reward-to-cost ratio from 1 to 50, in each case considering movement costs 0.1, 0.5, and 0.9 only. In all cases (except for resident defectors), staying propensity decreases over the range of reward-to-cost ratios because receiving larger reward of cooperation partially offsets potential movement costs. As we would expect, cooperators do better relative to defectors when the ratios are high. In particular in both figures we see that selection favours cooperators for sufficiently high ratios, and selection favours defectors for sufficiently low ratios, with a range of intermediate values (of increasing width with movement cost) where selection opposes change.

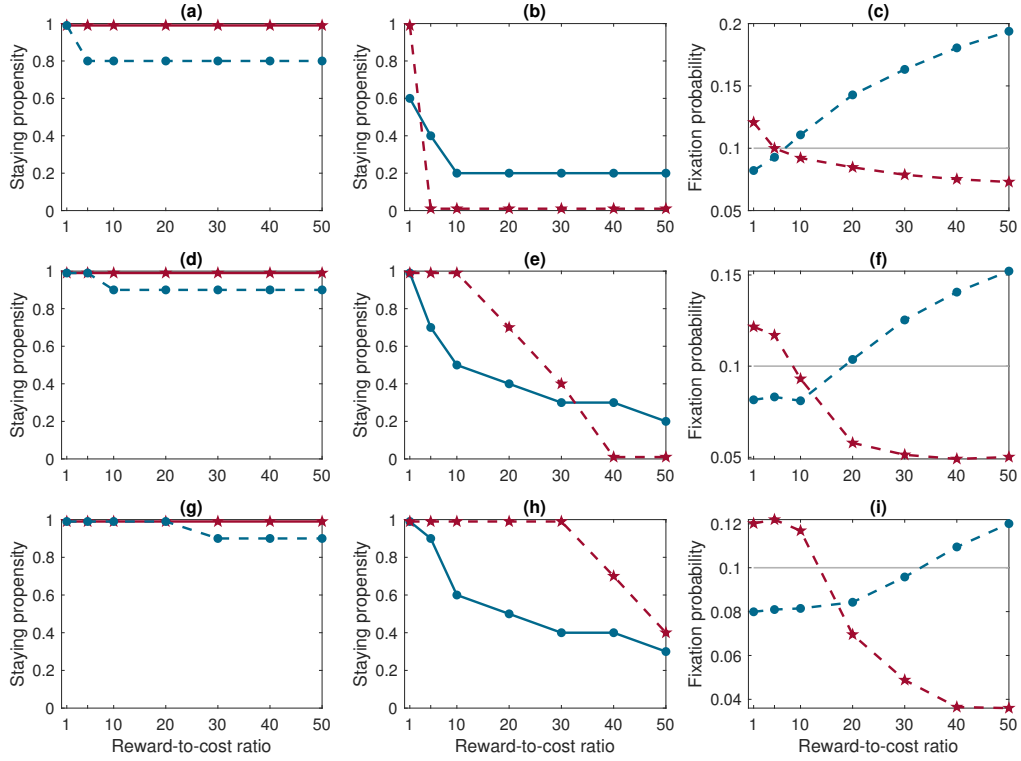


Figure 4. Varying the reward-to-cost ratio v/c for the complete graph, population size $N = 10$: (a)–(c) low movement cost $\lambda = 0.1$; (d)–(f) medium movement cost $\lambda = 0.5$; (g)–(i) high movement cost $\lambda = 0.9$. The ratio $v/c = 10$ corresponds to the base case. For legend see the figure 1 caption.

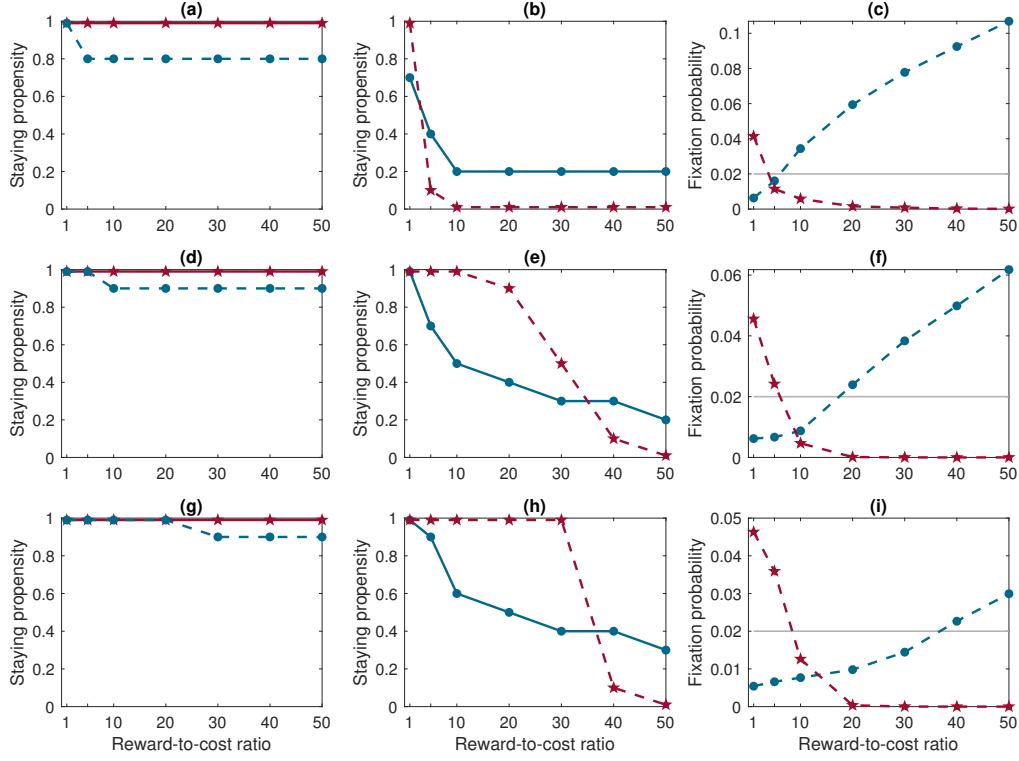


Figure 5. Varying the reward-to-cost ratio v/c for the complete graph, population size $N = 50$: (a)–(c) low movement cost $\lambda = 0.1$; (d)–(f) medium movement cost $\lambda = 0.5$; (g)–(i) high movement cost $\lambda = 0.9$. For legend see the figure 1 caption.

1.2. Circle graph. In figure 6 we consider the optimal staying propensities and fixation probabilities over the full range of population sizes, as we did for the complete graph in figure 1. The staying propensities follow the pattern similar to the complete graph case. There is generally precisely one dominant interactive strategy (except in the case for $N = 10$), as we see from the fixation probability lines crossing when both achieve the neutral value. Selection favours cooperators for movement costs below a threshold ($\lambda = 0.4$), and selection favours defectors for movement costs above the threshold.

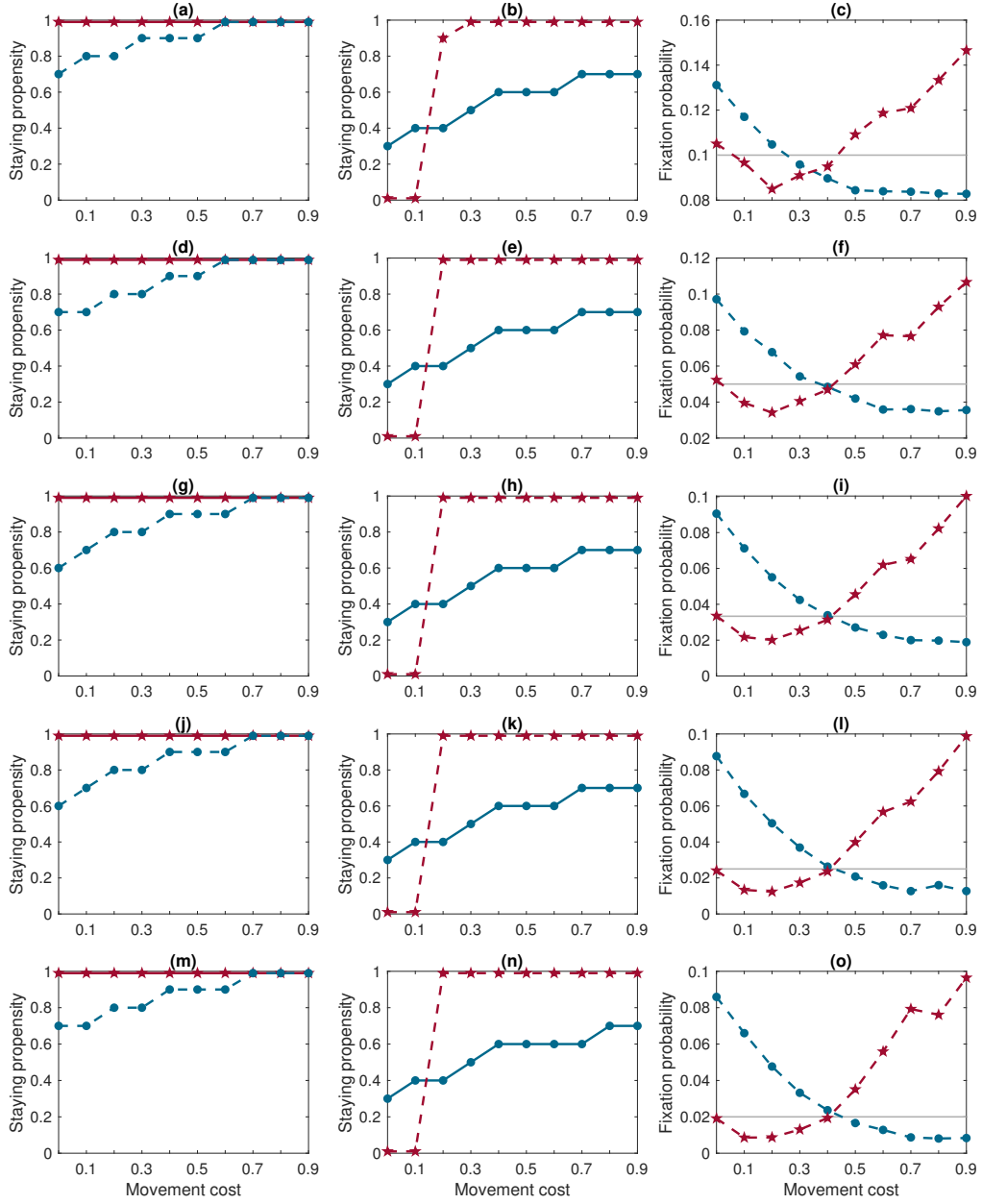


Figure 6. Varying the population size for the circle graph: (a)–(c) population size 10; (d)–(f) population size 20; (g)–(i) population size 30; (j)–(l) population size 40; (m)–(o) population size 50. For legend see the figure 1 caption.

In figures 7 (exploration time $T = 5$) and 8 (exploration time $T = 25$) we investigate changes in the exploration time for the circle graph. The staying propensities again behave as for the complete graph. Usually there is one dominant strategy: cooperators for longer exploration time and defectors for shorter exploration time.

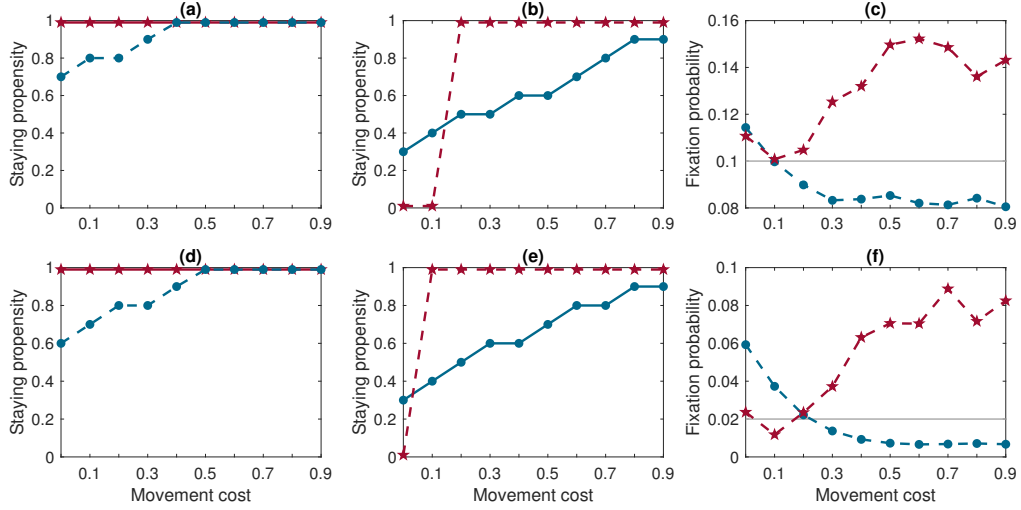


Figure 7. Short exploration time $T = 5$ for the circle graph: (a)–(c) population size 10; (d)–(f) population size 50. For legend see the figure 1 caption.

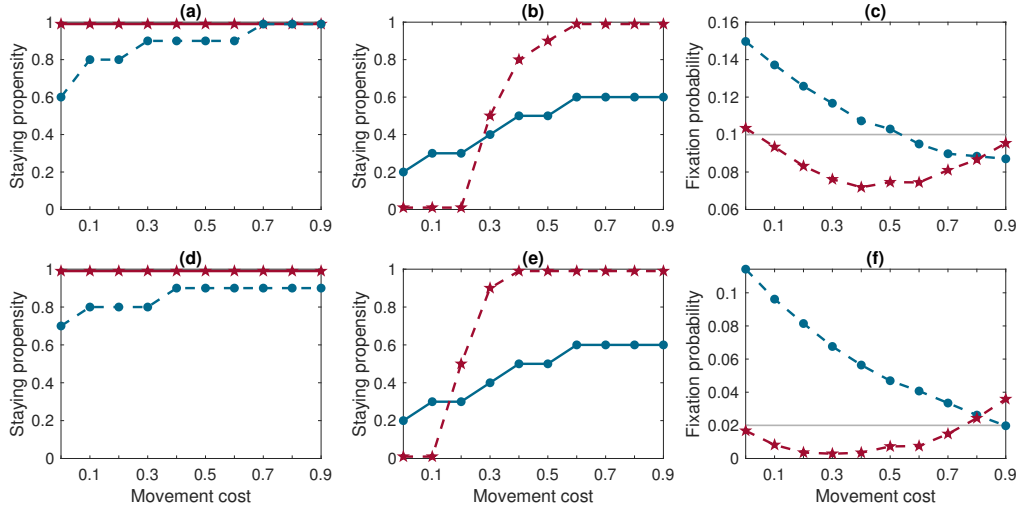


Figure 8. Long exploration time $T = 25$ for the circle graph: (a)–(c) population size 10; (d)–(f) population size 50. For legend see the figures 1 caption.

In figures 9 and 10 we vary the reward-to-cost ratio for population sizes $N = 10$ and $N = 50$ for movement costs 0.1, 0.5, and 0.9. We again observe that generally one strategy is dominant: cooperators if the reward-to-cost ratio is sufficiently large, and defectors if the reward-to-cost ratio is sufficiently small.

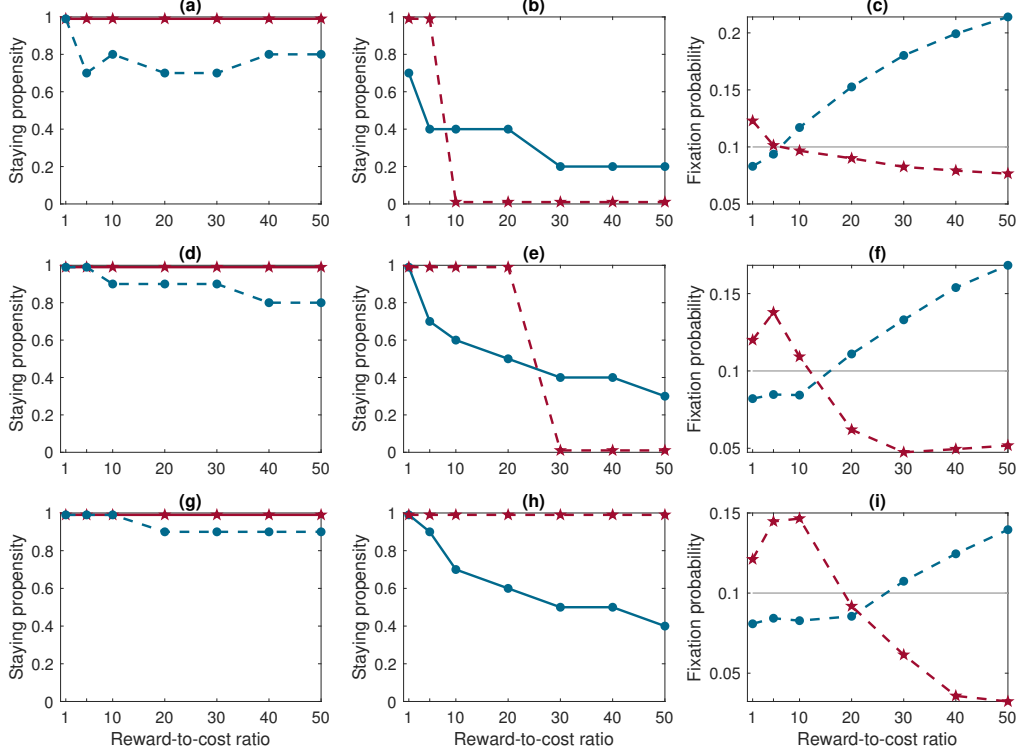


Figure 9. Varying the reward-to-cost ratio v/c for the circle graph, population size $N = 10$: (a)–(c) low movement cost $\lambda = 0.1$; (d)–(f) medium movement cost $\lambda = 0.5$; (g)–(i) high movement cost $\lambda = 0.9$. The ratio $v/c = 10$ corresponds to the base case. For legend see the figure 1 caption.

1.3. Star graph. In figure 11 we consider optimal staying propensities and fixation probabilities for various population sizes on our third structure, the star graph. Mutant cooperators have much lower staying propensity than on the complete and circle graphs. In the hub-and-spoke topology of the star graph, a cluster of cooperators is most likely to form in the hub (the centre of the star), and hence it is beneficial to get from the leaves to the centre quickly. For this same reason, resident cooperators evolve to the lowest staying propensity ($\lambda = 0.01$) for all movement costs and population sizes. Mutant defectors do best by imitating the strategy “get to the centre quickly”, and they prefer to use low staying propensities most of the time. The fixation probabilities for cooperators follow a similar pattern to the other graphs, where there is a movement cost threshold below which the fixation probability of cooperators

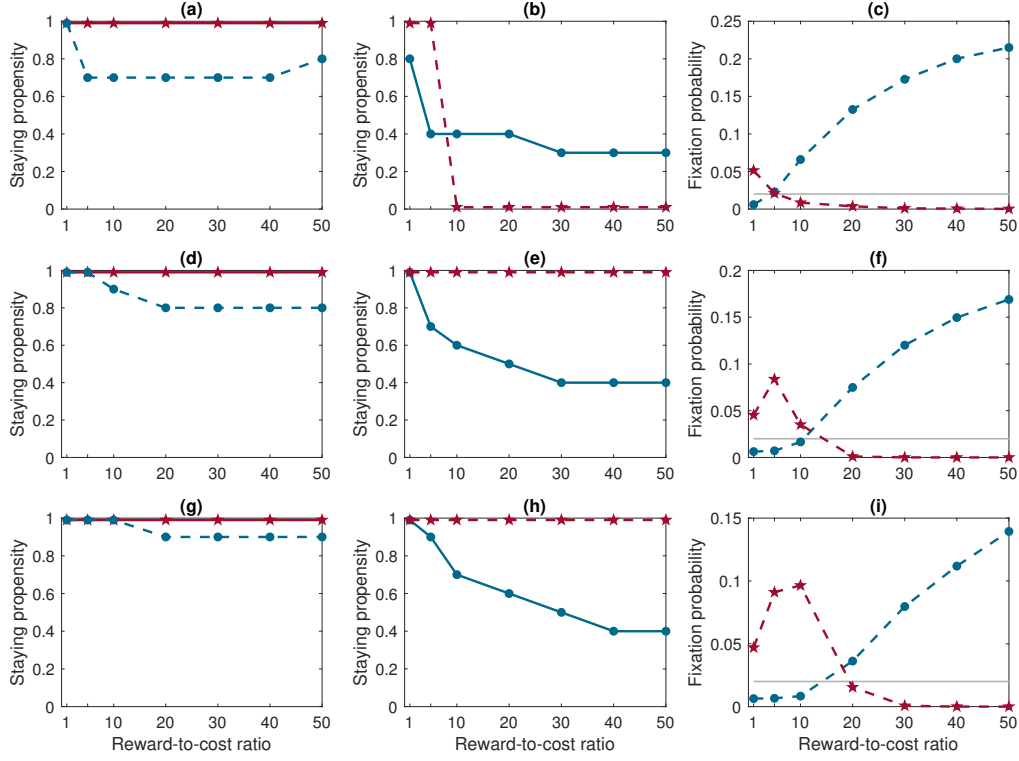


Figure 10. Varying the reward-to-cost ratio v/c for the circle graph, population size $N = 50$: (a)–(c) low movement cost $\lambda = 0.1$; (d)–(f) medium movement cost $\lambda = 0.5$; (g)–(i) high movement cost $\lambda = 0.9$. The ratio $v/c = 10$ corresponds to the base case. For legend see the figure 1 caption.

exceeds the neutral drift one; this threshold increases with the population size. The most striking feature of figure 11, however, is that for all combinations of population size and movement cost, the fixation probability of mutant defectors always exceeds the neutral drift one. It is easy for defectors to locate and exploit the cooperating cluster in the centre of the star.

In figures 12 and 13 we consider shorter and longer exploration times for population sizes 10 and 50. There is not a large effect on the staying propensities, except that the mutant defectors choose a higher staying propensity than before when the exploration time is longer, in figure 13. Interestingly both shortening the exploration time and extending the exploration time makes it more difficult for mutant cooperators to replace resident defectors. Thus intermediate values of the searching time seem optimal for cooperator invaders. On the one hand, cooperators need to spend sufficient time together to accumulate the rewards; hence shortening the exploration time hurts cooperators. On the other hand, in the hub-and-spoke topology of the star graph, slow-moving

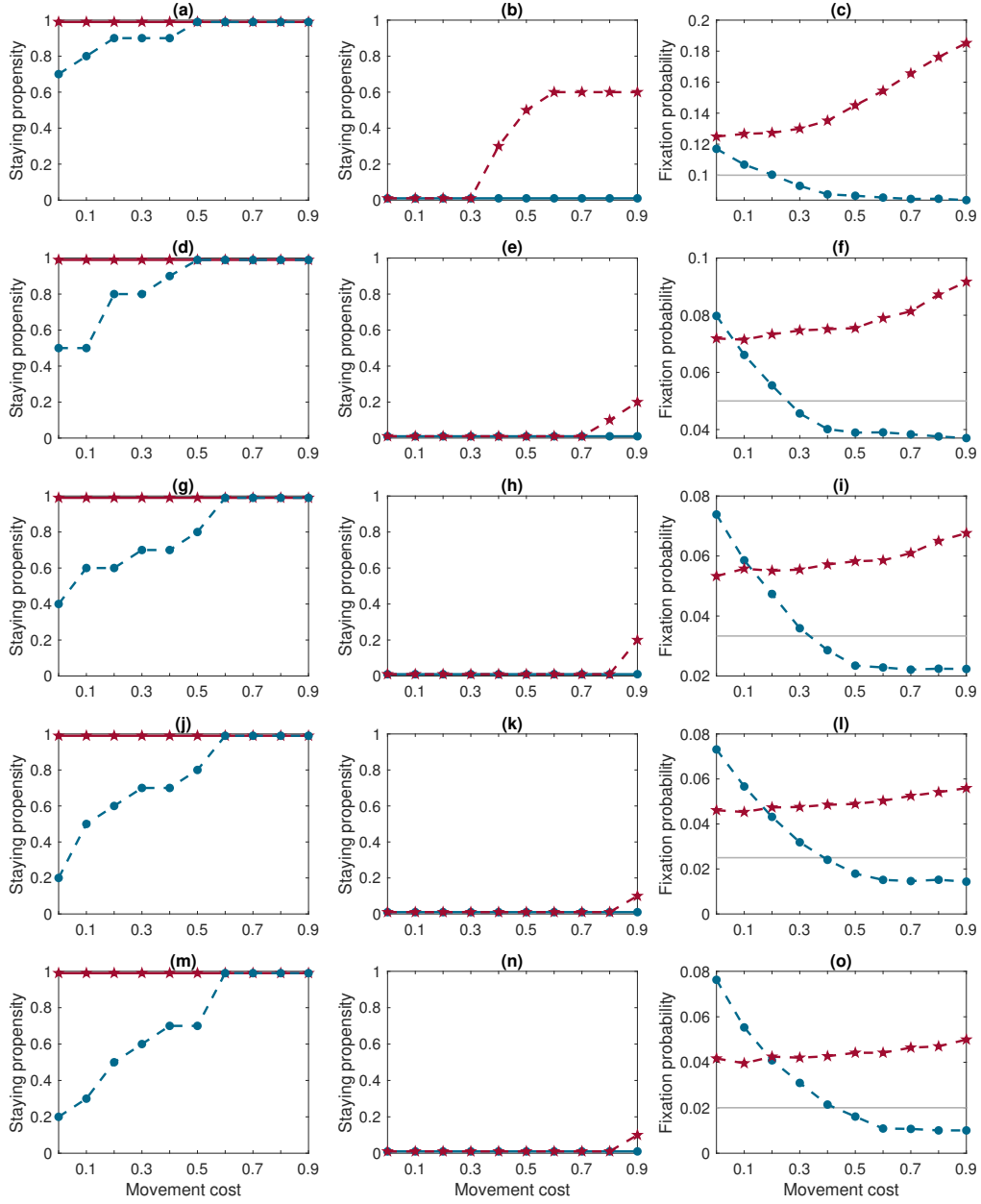


Figure 11. Varying the population size for the star graph: (a)–(c) population size 10; (d)–(f) population size 20; (g)–(i) population size 30; (j)–(l) population size 40; (m)–(o) population size 50. For legend see the figure 1 caption.

resident defectors (with staying propensity $\lambda = 0.99$) are more likely to eventually move to the centre and thus exploit a potential cooperating cluster there. So, too long an exploration phase also hurts cooperators.

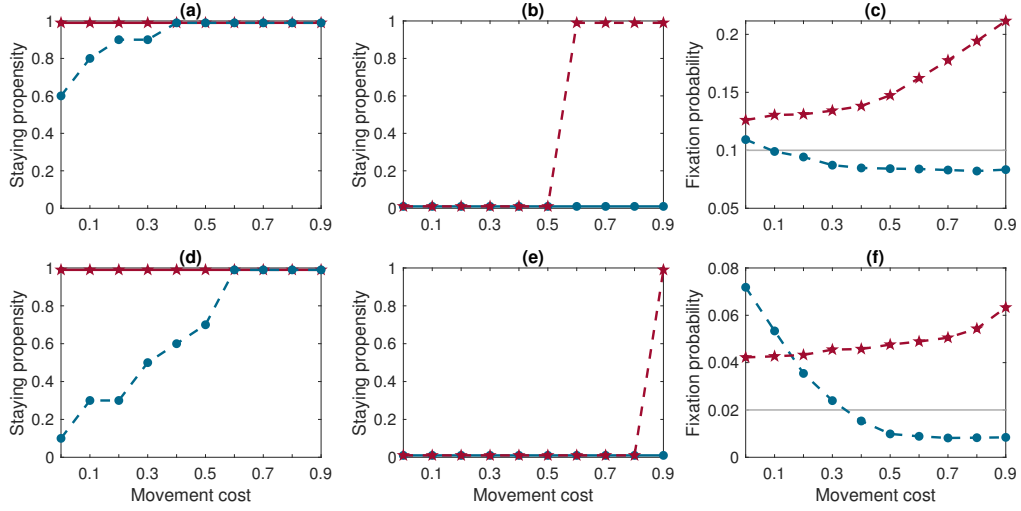


Figure 12. Short exploration time $T = 5$ for the star graph: (a)–(c) population size 10; (d)–(f) population size 50. For legend see the figure 1 caption.

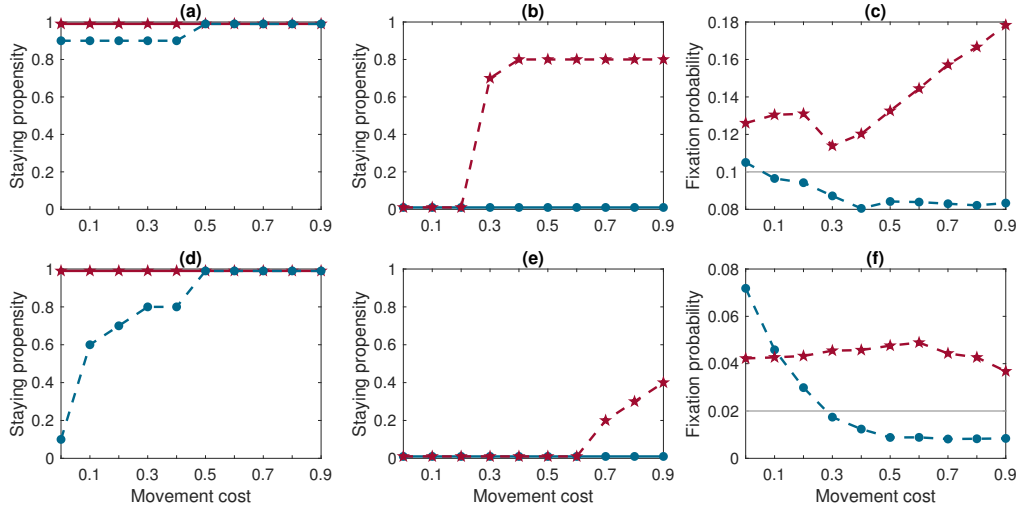


Figure 13. Long exploration time $T = 25$ for the star graph: (a)–(c) population size 10; (d)–(f) population size 50. For legend see the figure 1 caption.

In figures 14 and 15 we vary the reward-to-cost ratio for the star graph with population sizes 10 and 50 for movement costs 0.1, 0.5, and 0.9. The equilibrium staying propensity of resident cooperators increases to the maximum value 0.99 when the reward-to-cost ratio is smallest ($v/c = 1$) and the movement cost is sufficiently high. In this case, the low value of the reward of cooperation does not compensate the potentially high cost of movement. As before, the fixation probability of mutant defectors always exceeds the neutral drift one. Mutant cooperators can replace resident defectors for sufficiently high reward-to-cost ratio for any movement cost.

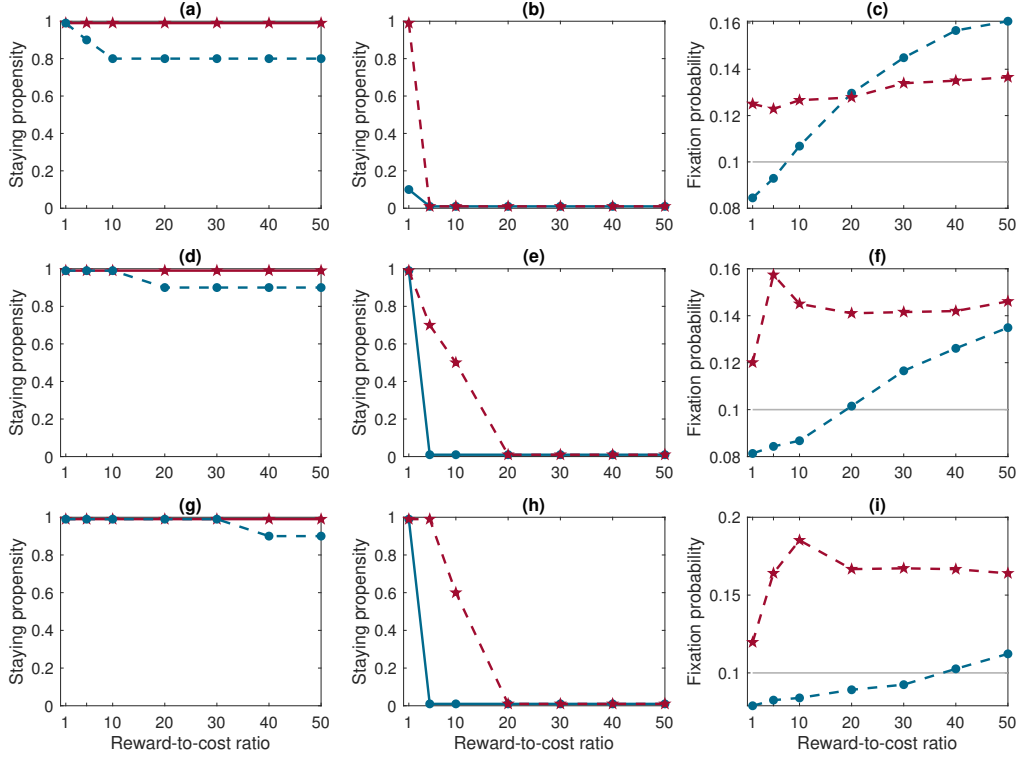


Figure 14. Varying the reward-to-cost ratio v/c for the star graph, population size $N = 10$: (a)–(c) low movement cost $\lambda = 0.1$; (d)–(f) medium movement cost $\lambda = 0.5$; (g)–(i) high movement cost $\lambda = 0.9$. The ratio $v/c = 10$ corresponds to the base case. For legend see the figure 1 caption.

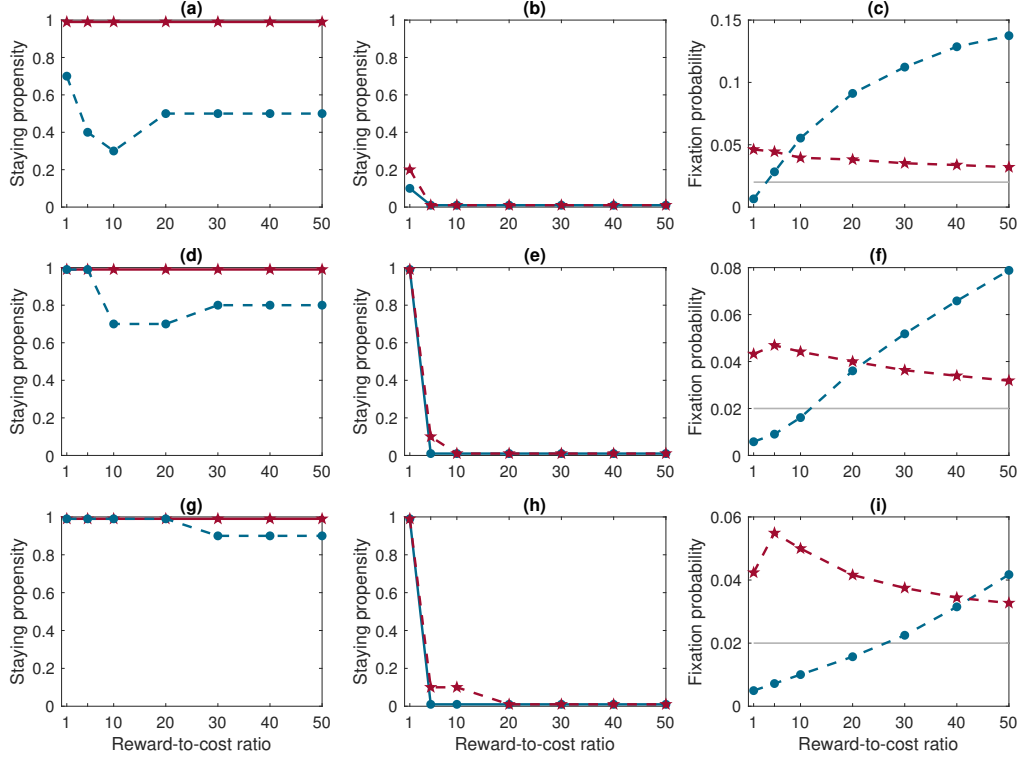


Figure 15. Varying the reward-to-cost ratio v/c for the star graph, population size $N = 50$: (a)–(c) low movement cost $\lambda = 0.1$; (d)–(f) medium movement cost $\lambda = 0.5$; (g)–(i) high movement cost $\lambda = 0.9$. The ratio $v/c = 10$ corresponds to the base case. For legend see the figure 1 caption.

2. SCENARIO 2: INTERACTIVE STRATEGY MUTATIONS ARE NOT RARE

In this scenario, we only consider two populations sizes (10 and 50), and we vary the exploration time and the reward-to-cost ratio. Similarly to the scenario 1, we group the results by the network structure.

2.1. Complete graph. Figure 16 shows the equilibrium staying propensities for cooperators and defectors in a mixed (half-and-half) population with normal exploration time ($T = 10$), the fixation probabilities of cooperators and defectors using the equilibrium staying propensities in a mixed population, and the fixation probabilities of cooperator and defector mutants in a defector and cooperator resident population, respectively, where both mutants and residents inherit staying propensities from the mixed population. Defectors evolve their staying propensity to the lowest possible one (0.01) for low movement costs and the highest possible one (0.99) for high movement costs; they rarely utilize intermediate staying propensities. In contrast, cooperators never evolve to the extreme staying propensities; their equilibrium staying propensity increases from 0.2 to 0.9 with the movement cost.

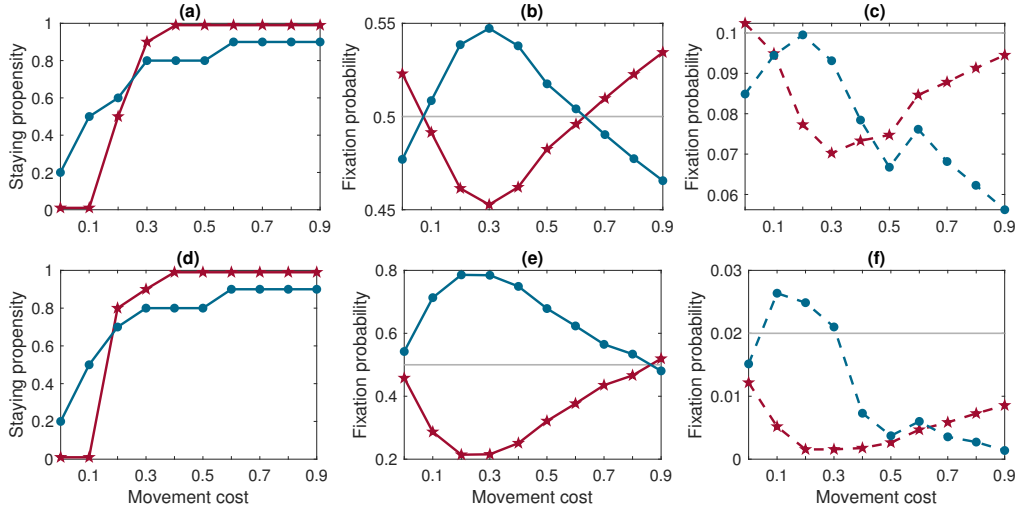


Figure 16. Non-rare interactive mutations case for the complete graph: (a)–(c) population size $N = 10$; (d)–(f) population size $N = 50$. The left-column panels show mutually best staying propensities for cooperators and defectors in a mixed population, the middle-column panels show fixation probabilities of cooperators and defectors with mutually best staying propensities in a mixed population, and the right-column panels show the fixation probabilities of mutant cooperators and mutant defectors in a resident defector and cooperator populations, respectively, where both cooperators and defectors inherit staying propensities from the mixed population. Legend: defectors—red with star markers for data points, cooperators—blue with round markers for data points; mixed population—solid line, mutants—dashed line.

In a small mixed population, cooperators do better than defectors for intermediate movement costs (0.1–0.6); defectors prevail otherwise. In a large mixed population, cooperators dominate defectors for all movement costs except the highest one (0.9). The mutant-residents population shares the basic qualitative behavior with scenario 1: selection usually opposes change. In a small mutant-residents population, selection opposes change for all positive movement costs. In a large mutant-residents population, selection favours cooperators for movement costs 0.1–0.3 and opposes change otherwise. In general, cooperators do better in larger populations on the complete graph.

Figures 17 and 18 show the effect of the exploration time. Shortening the exploration time significantly hurts cooperators and extending the exploration time significantly helps cooperators in both mixed and mutant-residents populations.

With a short exploration phase ($T = 5$), defectors do better than cooperators in a small mixed population for all movement costs except 0.2, and defectors dominate cooperators in a large mixed population for all movement costs above 0.3. In a mutant-residents population, selection favours defectors for sufficiently high movement cost and for zero movement cost in a small population. The fixation probability of mutant cooperators never exceeds the neutral drift one if the exploration time is too short.

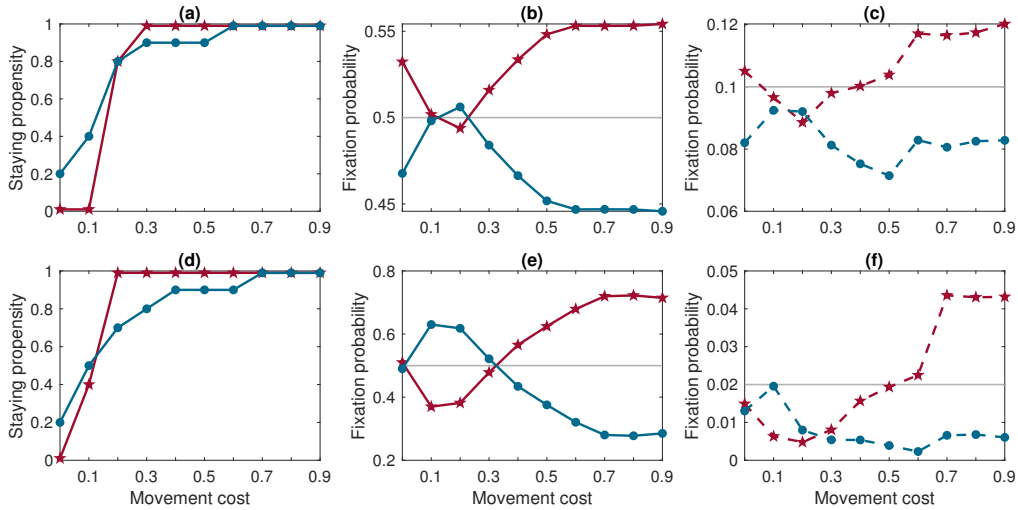


Figure 17. Short exploration time $T = 5$ in the non-rare interactive mutations case for the complete graph: (a)–(c) population size 10; (d)–(f) population size 50. For legend see the figure 16 caption.

With a long exploration phase ($T = 25$), cooperators dominate defectors in a mixed population in all cases except for movement cost 0 in a small population. In a mutant-resident population, selection favour cooperators for intermediate

movement costs, and selection favours defectors only in for movement cost 0 in a small population.

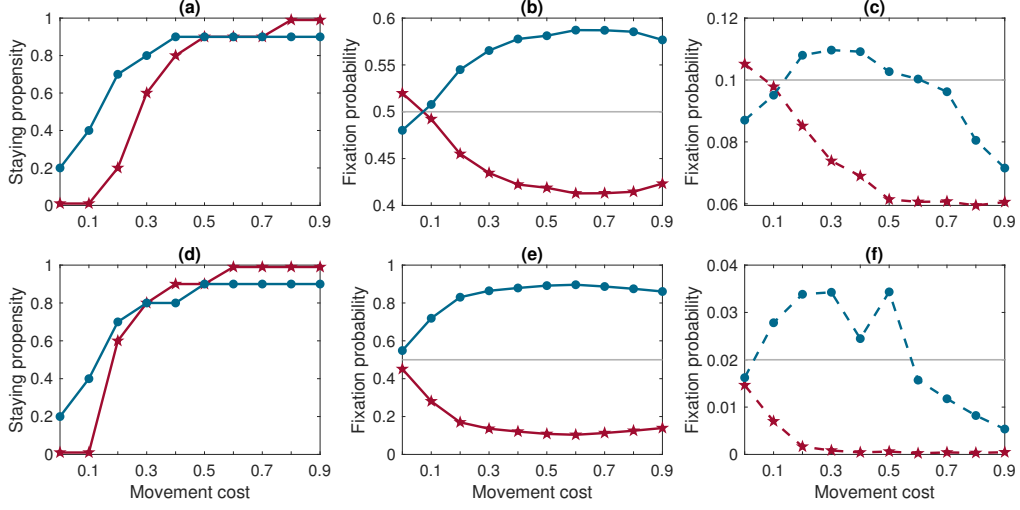


Figure 18. Long exploration time $T = 25$ in the non-rare interactive mutations case for the complete graph: (a)–(c) population size 10; (d)–(f) population size 50. For legend see the figure 16 caption.

Figures 19 and 20 explore the effect of varying the reward-to-cost ratio for movement costs 0.1, 0.5, and 0.9.

If the reward-to-cost ratio is too small then defectors dominate cooperators in all population types, and if the ratio is sufficiently large then cooperators dominate defectors in a mixed population. Interestingly, selection never favours cooperators in a mutant-residents small population for low movement cost ($\lambda = 0.1$). In this case, resident defectors inherit very low staying propensity from the mixed population, and hence they are better suited to resist mutants in a low-movement-cost environment.

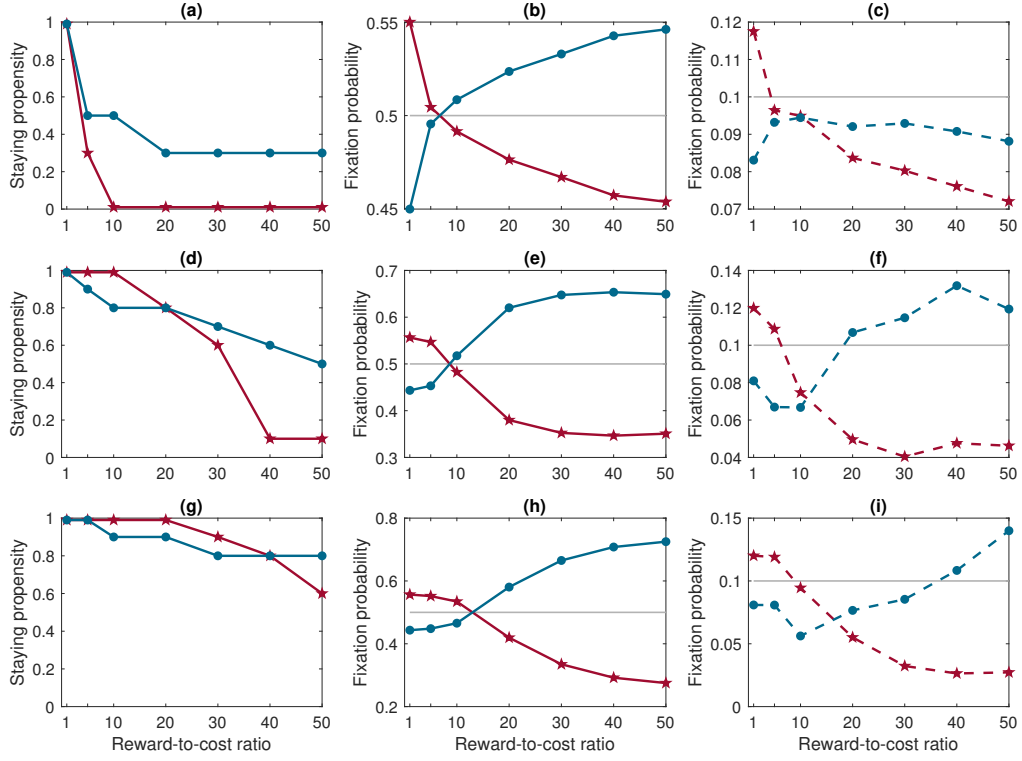


Figure 19. Varying the reward-to-cost ratio v/c in the non-rare interactive mutations case for the complete graph, population size $N = 10$: (a)–(c) low movement cost $\lambda = 0.1$; (d)–(f) medium movement cost $\lambda = 0.5$; (g)–(i) high movement cost $\lambda = 0.9$. The ratio $v/c = 10$ corresponds to the base case. For legend see the figure 16 caption.

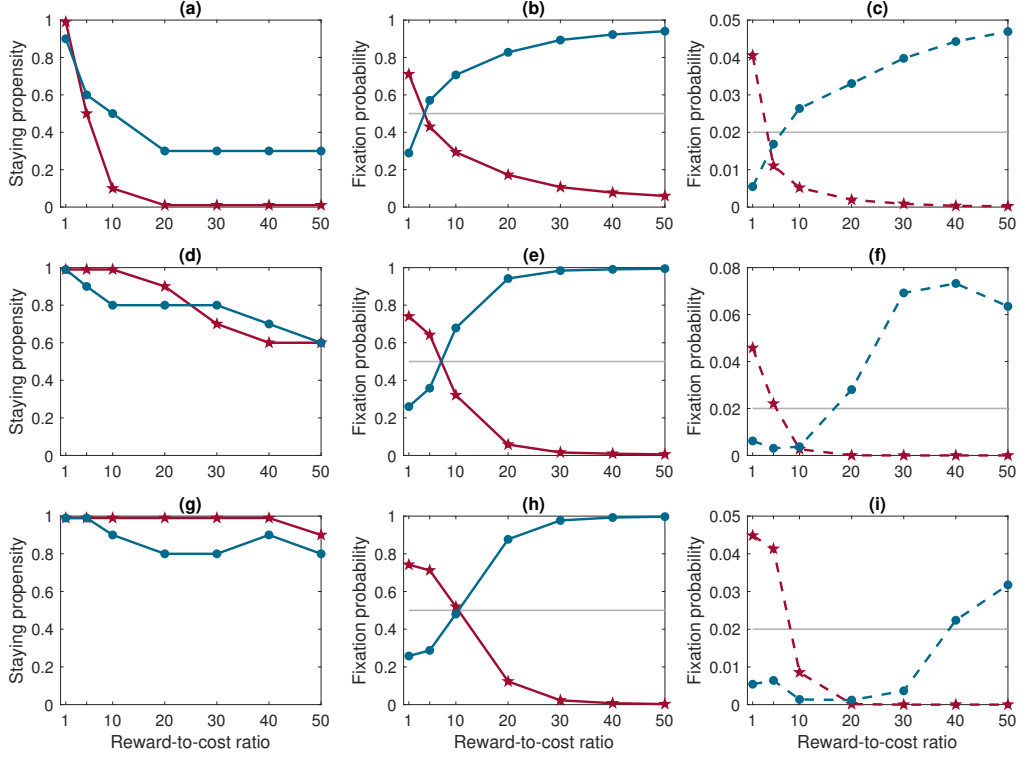


Figure 20. Varying the reward-to-cost ratio v/c in the non-rare interactive mutations case for the complete graph, population size $N = 50$: (a)–(c) low movement cost $\lambda = 0.1$; (d)–(f) medium movement cost $\lambda = 0.5$; (g)–(i) high movement cost $\lambda = 0.9$. The ratio $v/c = 10$ corresponds to the base case. For legend see the figure 16 caption.

2.2. Circle graph. Figure 21 presents the base case for the circle graph. Similarly to the complete graph, defectors rarely evolve to intermediate staying propensities in a mixed population, and cooperators do the opposite: they rarely use extreme staying propensities. In a mixed population, cooperators do better than defectors for intermediate movement costs 0.1–0.4. Defectors do better than cooperators for all other movement costs except in a large population where they are tied with cooperators for movement costs 0 and 0.5. In a large mutant-residents population, the situation is similar to the mixed population case: selection favours cooperators for intermediate movement costs, selection favours defectors for high movement costs, and selection opposes change at two threshold movement cost values 0 and 0.5. In a small mutant-residents population, selection favours cooperators only for movement cost 0.2. Selection favours defectors for zero and high movement costs, and selection opposes change otherwise.

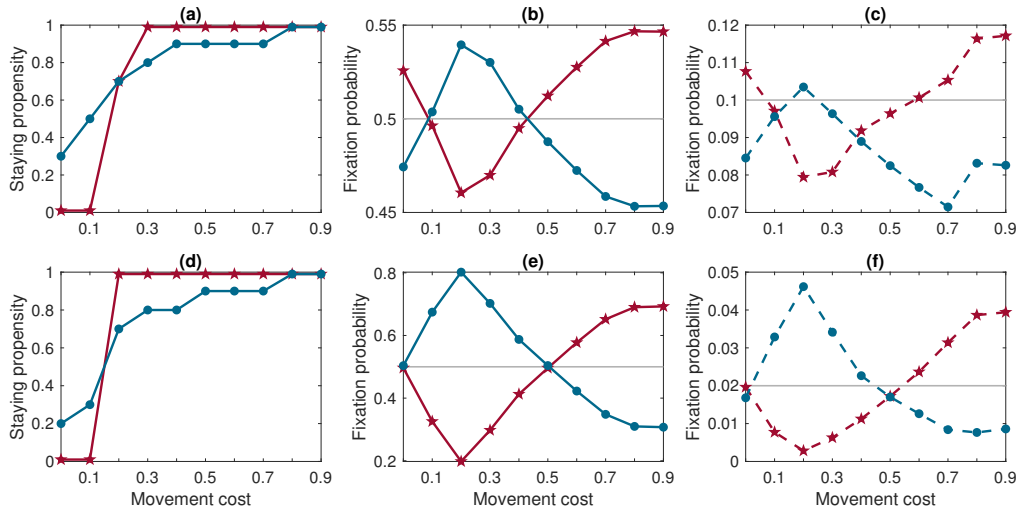


Figure 21. Non-rare interactive mutations case for the circle graph: (a)–(c) population size $N = 10$; (d)–(f) population size $N = 50$. For legend see the figure 16 caption.

Figures 22 and 23 investigate the effect of the exploration time. The situation is similar to the complete graph: shortening the exploration time hurts cooperators, while extending the exploration time helps cooperators. With the short exploration phase ($T = 5$), cooperators never do better than defectors in a small mixed population. With the long exploration phase ($T = 25$), defectors never do better than cooperators in a large mixed population. The fixation probabilities of mutant cooperators and mutant defectors are always below the neutral drift one with the short exploration phase in a small population and with the long exploration phase in a large population, respectively.

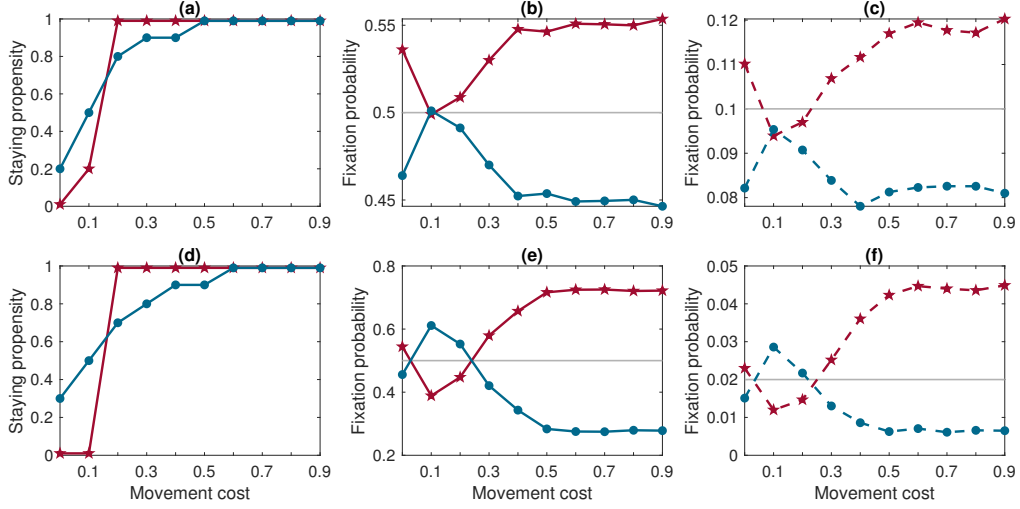


Figure 22. Short exploration time $T = 5$ in the non-rare interactive mutations case for the circle graph: (a)–(c) population size 10; (d)–(f) population size 50. For legend see the figure 16 caption.

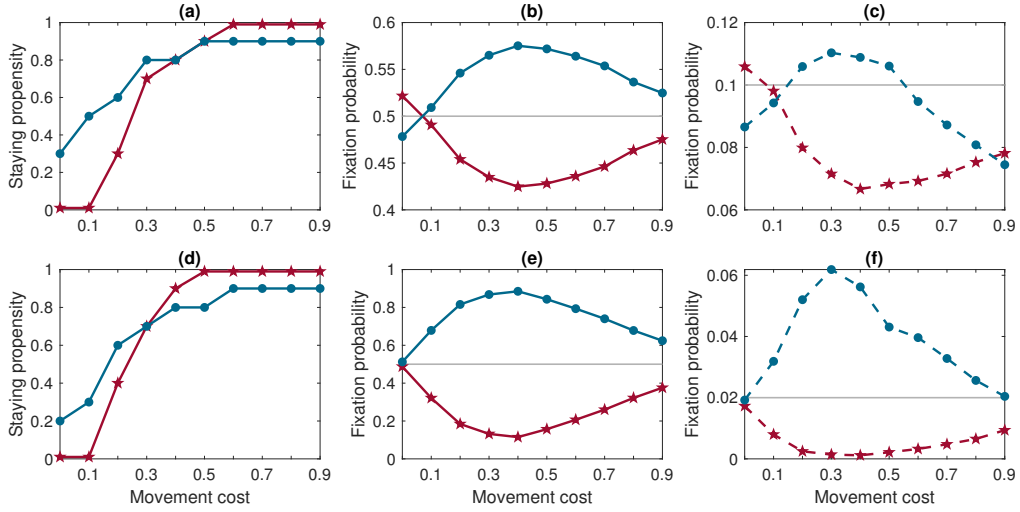


Figure 23. Long exploration time $T = 25$ in the non-rare interactive mutations case for the circle graph: (a)–(c) population size 10; (d)–(f) population size 50. For legend see the figure 16 caption.

Figures 24 and 24 demonstrate the results for various reward-to-cost ratios. The results are similar to the complete graph case: defectors dominate over the entire range of movement cost values if the reward-to-cost ratio is sufficiently small, and cooperators dominate over the entire range of movement cost values if the reward-to-cost ratio is sufficiently high.

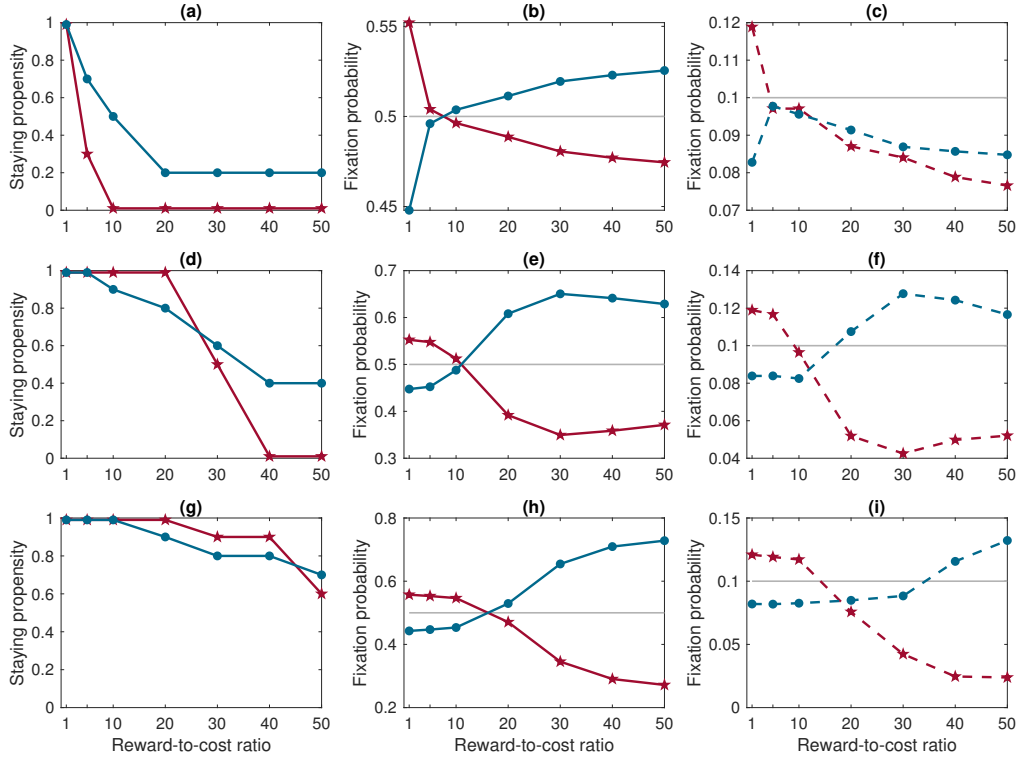


Figure 24. Varying the reward-to-cost ratio v/c in the non-rare interactive mutations case for the circle graph, population size $N = 10$: (a)–(c) low movement cost $\lambda = 0.1$; (d)–(f) medium movement cost $\lambda = 0.5$; (g)–(i) high movement cost $\lambda = 0.9$. The ratio $v/c = 10$ corresponds to the base case. For legend see the figure 16 caption.

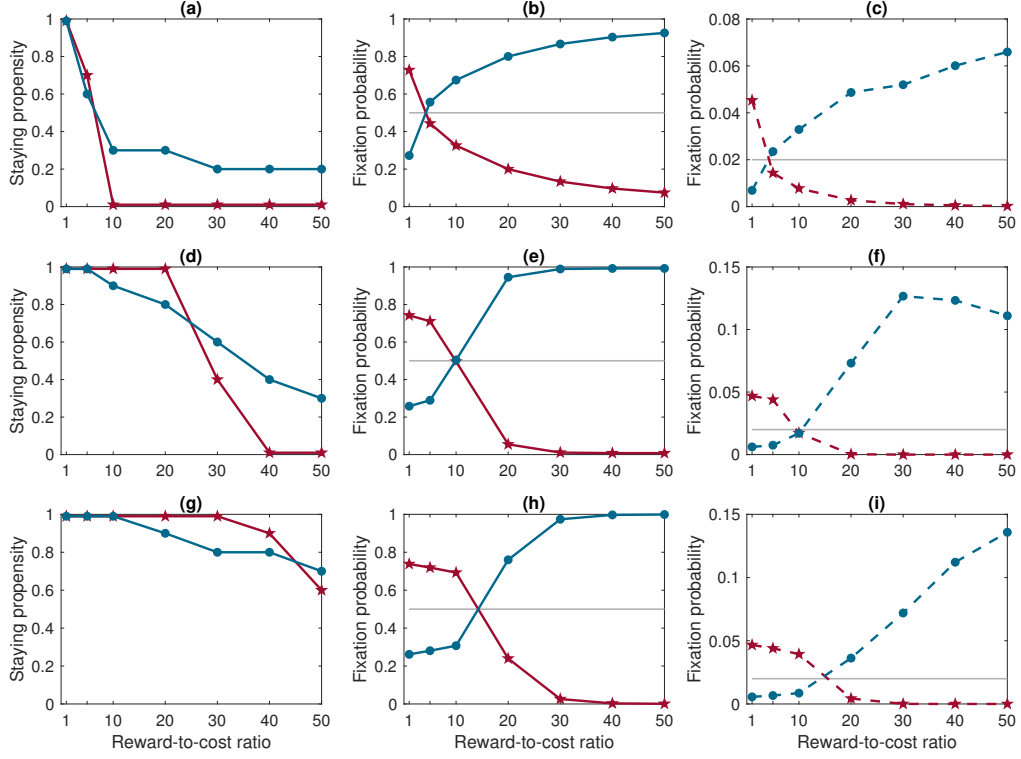


Figure 25. Varying the reward-to-cost ratio v/c in the non-rare interactive mutations case for the circle graph, population size $N = 50$: (a)–(c) low movement cost $\lambda = 0.1$; (d)–(f) medium movement cost $\lambda = 0.5$; (g)–(i) high movement cost $\lambda = 0.9$. The ratio $v/c = 10$ corresponds to the base case. For legend see the figure 16 caption.

2.3. Star graph. The star graph presented a unique challenge for the mixed population case: there were no equilibrium staying propensities for cooperators and defectors in a large mixed population for movement costs above 0.1. See the main text for an explanation and our proposed resolution.

Figure 26 presents the base case results. In a mixed population, cooperators do slightly better than defectors for a range of intermediate movement costs; defectors do better than cooperators for low and high movement costs. In a mutant-residents population, the fixation probability of cooperators never exceeds the neutral drift one. Selection favours defectors for most values of the movement cost. Overall, defectors do better than cooperators on the star graph even in the non-rare interactive mutations case.

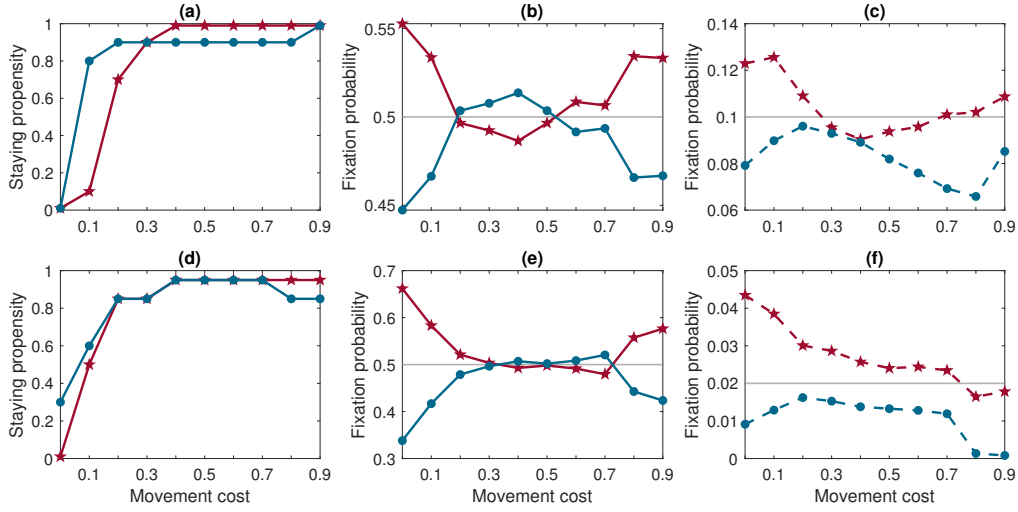


Figure 26. Non-rare interactive mutations case for the star graph: (a)–(c) population size $N = 10$; (d)–(f) population size $N = 50$. For legend see the figure 16 caption.

Figures 27 and 28 investigate various exploration times. Shortening the exploration phase to $T = 5$ results in complete dominance of defectors for all other parameter combinations. Extending the exploration phase to $T = 25$ finally allows cooperators to dominate defectors in a mixed population over all but lowest movement costs. In a mutant-residents population, mutant defectors can replace residents cooperators only for movement costs 0–0.2, and mutant cooperators can replace resident defectors for movement costs 0.1–0.6 in a large population.

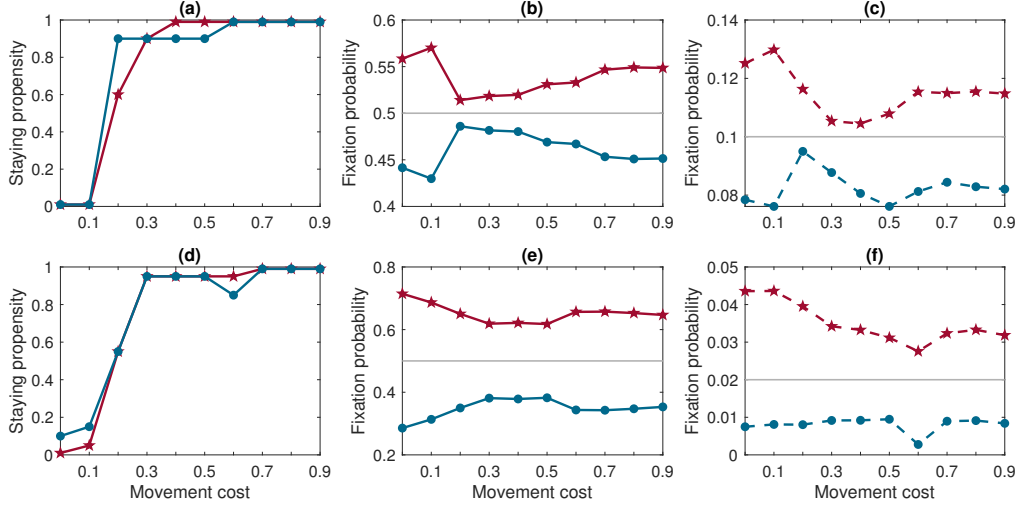


Figure 27. Short exploration time $T = 5$ in the non-rare interactive mutations case for the star graph: (a)–(c) population size 10; (d)–(f) population size 50. For legend see the figure 16 caption.

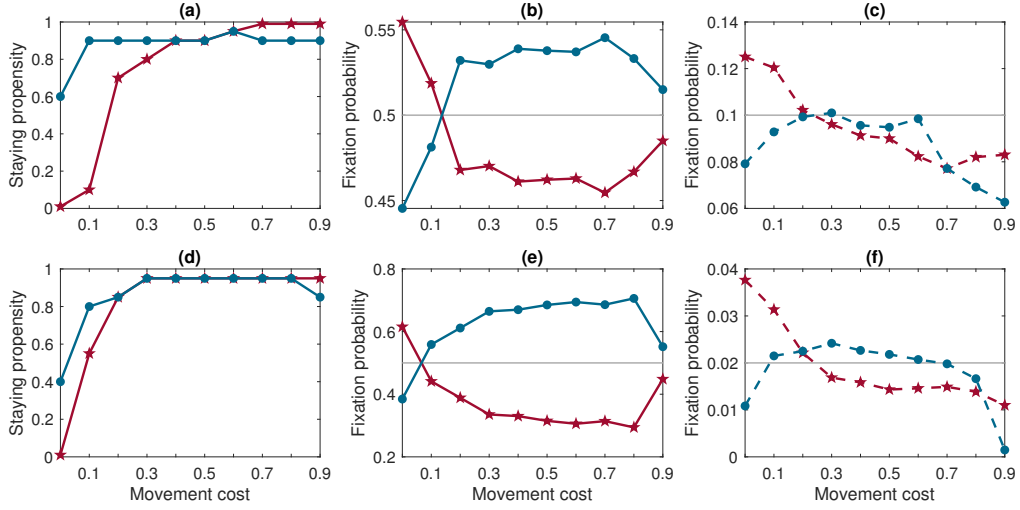


Figure 28. Long exploration time $T = 25$ in the non-rare interactive mutations case for the star graph: (a)–(c) population size 10; (d)–(f) population size 50. For legend see the figure 16 caption.

Figures 29 and 30 summarise the outcomes for varying the reward-to-cost ratio. This ratio has no effect in small populations and low movement cost (0.1): defectors dominate cooperators in both mixed and mutant-residents populations. Otherwise, the outcomes for a mixed population aligns with other structures: defectors do better for small values of the reward-to-cost ratio, and cooperators do better for large values of the reward-to-cost ratio. Similar situation is observed in a mutant-resident population except in a small population for intermediate movement cost (0.5), where selection favours change for large values of the reward-to-cost ratio. In this parameter regime, defectors evolve to sufficiently low staying propensity (cf. figure 29d), while cooperators do not. Also, the fixation probability of mutant defectors always exceeds the neutral drift one in a large population and low movement cost (0.1) because defectors evolve to lower staying propensities than cooperators for high reward-to-cost ratio in a mixed population (cf. figure 30a).

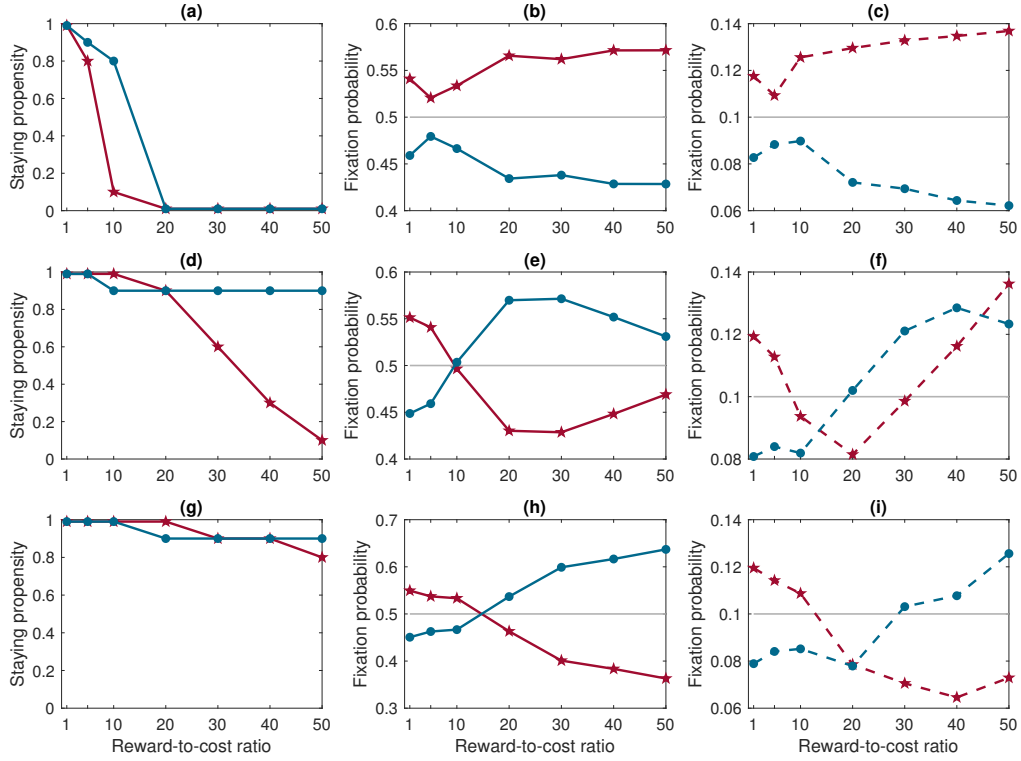


Figure 29. Varying the reward-to-cost ratio v/c in the non-rare interactive mutations case for the star graph, population size $N = 10$: (a)–(c) low movement cost $\lambda = 0.1$; (d)–(f) medium movement cost $\lambda = 0.5$; (g)–(i) high movement cost $\lambda = 0.9$. The ratio $v/c = 10$ corresponds to the base case. For legend see the figure 16 caption.

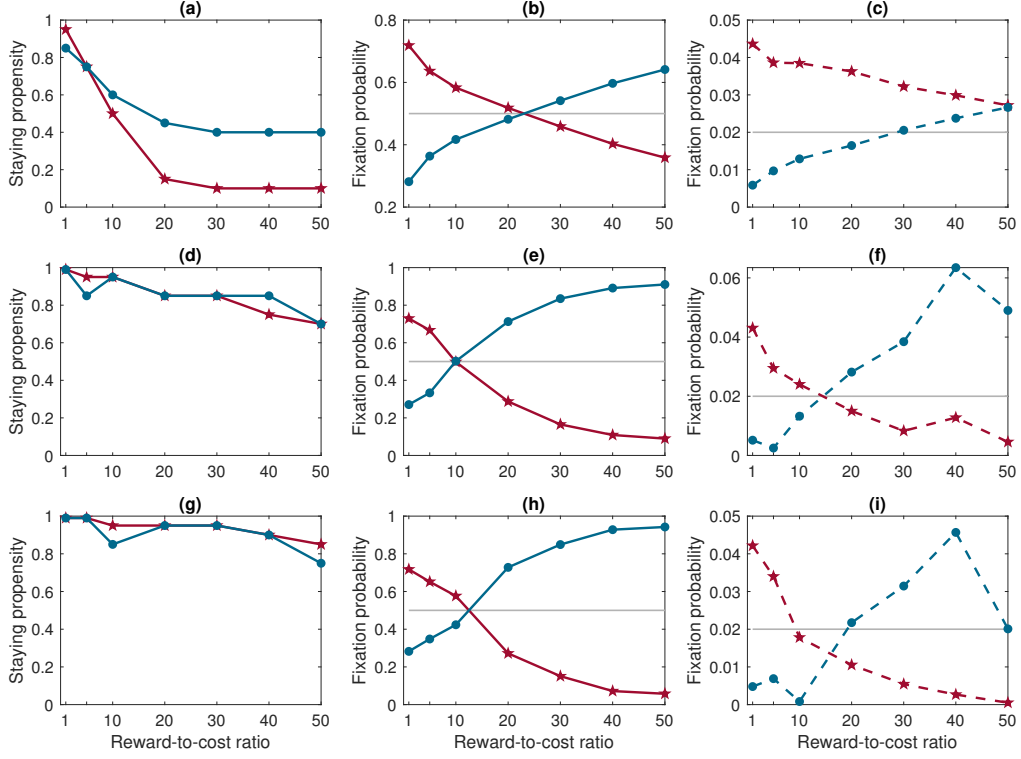


Figure 30. Varying the reward-to-cost ratio v/c in the non-rare interactive mutations case for the star graph, population size $N = 50$: (a)–(c) low movement cost $\lambda = 0.1$; (d)–(f) medium movement cost $\lambda = 0.5$; (g)–(i) high movement cost $\lambda = 0.9$. The ratio $v/c = 10$ corresponds to the base case. For legend see the figure 16 caption.