**Supplementary Materials to paper**

**Generalized Fock space and contextuality**

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These Supplementary materials contain some details of the derivation of the equations and explanation of some results which is not included in the paper due to limitations in the size of the paper.

**3. Arbitrary operators in Fock space**

An arbitrary operator (27) can be expressed in terms of the operator .

In fact, according to (5) and (6) one obtains , or, taking into account (14), . This expression can be rewritten in the form , where is the arbitrary constant. Then, assuming , one obtains . Using such a representation of the operator , we fix the form of the function . Moreover, the function can be arbitrary. Note that in this way, one can express only one of the set of operators of type (27) defined in the space . All other operators of type (27) in the same space with a fixed function can also be expressed in terms of the raising and lowering operators, but this connection will be more complex and can be represented as a function , where is some function that can be represented as a power series by its arguments.

We consider here a particular case: the representation of the operators (28) through rising and lowering operators. We assume that the eigenvalues of the operator can be written as a function of the integer : . In addition, we assume that the function can be expanded in a power series

 (A1)

Define the Fock space (up to an arbitrary function ) by fixing the function in the form . Then the operator of integer (20) defines the basis of the space , which are its eigenvectors:

 (A2)

Obviously,

 (A3)

where ; .

Thus, the basis is also eigenvectors (eigenfunctions) of the operators , while the parameters are their corresponding eigenvalues.

Taking into account (A1) and (A3), any operator (27) can be represented in this space as

 (A4)

which is easily verified by substituting (A4) into (27) by appealing to (A3) and (A1). Relation (A4) can be considered as a formal decomposition into a power series of an operator function

 (A5)

which is the representation of the operator through the operator of integer (20).

**4. Vector representation of random observables**

Taking into account (30), (1) and (28), the mean value of the parameter in the random process under consideration can be represented as . From (31) it follows that , where ; . By definition, . Taking into account (1), (3), (4) and (6), one can write . The mean value of a certain function , which depends on the parameter , in the considered random process is determined by the relation . Using a power series , one obtains . This expression can be rewritten in the form . As usual, we can introduce the formal operator . Then, one obtains .

Thus, any statistical characteristic of the random process can be calculated (at least formally) using the dual vectors (29) and the operator (31).

**5. Fock space and contextuality**

As for example, consider the superposition of states in the Doi-Peliti probability space (48). Suppose there are two non-correlated processes and which are realized with probabilities and , where . We assume that the phases of probabilities for these processes are equal . We can introduce the vectors

and

which describe the whole process. Note that in this case we can formally redefine the vectors : . This means that we can consider .

It is easy to verify that and . Indeed,

Thus, we see that in the Doi-Peliti probability space, see rule (48), for processes such that the phase parameters are equal; there is no interference of probabilities. This means that such a representation of the random processes does not take into account contextuality.