**Supplementary Information**

**1. Plane stress UMAT formulation**

The true stress of our self-healing hydrogel, assuming Gaussian chain statistics, is

. (S1)

The deformation gradients ,  are given by eqs.(2a) and (3b), and the Lagrange multiplier  is given by eq.(4). Thus, the in-plane stress components are

. (S2)

The in-plane components of the left Cauchy-Green tensor , and the out-of-plane stretch  can be calculated from the in-plane components of the deformation gradient .

Next, we use the same procedure as was described in [32] to approximate the relaxation function by a Prony series of  terms:

, (S3)

and rewrite the true stress as

. (S4)

where

, (S5a)

 (S5b)

 (S5c)

Eq. (S5a) can be readily calculated by the in-plane components of the current deformation gradient, . Eqs. (S5b-c) are updated based on the following scheme.  Let, and  be the in-plane components of , then . At :

 , (S6a)

. (S6b)

The last terms in eqs.(S6a,b) are obtained by approximating the integral from  to  using the trapezoidal rule. As is shown in eqs.(S6a,b), the updated state variables  and  depend only on , , and . The true stress at is updated by substituting eqs.(S6a,b) into eq.(S4).

**2. Convergence check for Figure 2b**

We simulate a Neo-Hookean solid with the geometry shown in Figure 2a and the mesh shown in Figure 2b, and compare both the crack opening displacement  and the dominant stress  with known asymptotic results given in [30,31], i.e.,

, , as . (S7)

The FEM results corresponding to an applied nominal stretch ratio of are shown in Figure S1. This figure shows that excellent agreement between asymptotic and FEM results, confirming that the mesh we use in our FE simulations can correctly capture the crack tip fields.

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| (a) | (b) |
| Figure S1. Comparisons between plane stress FEM and asymptotic fields of a Neo-Hookean solid to check the mesh. The geometry is given by Figure 2a and the mesh is given by Figure 2b. The crack opening displacement and the true stress are taken at a nominal stretch ratio . | |

**3. Comparison with DIC: crack tip with finite size of radius**

The sample used in the DIC experiment is cutted by an X-ACTO blade which has a finite tip radius, as a result, the crack in the DIC sample is not perfectly sharp. This local crack blunting affect the local strain field. To account for this effect, we carry out an FE simulation with a notched crack specimen where the crack tip has a finite radius of 0.1mm (see Figure S2). The FE Lagrangian strain fields for this specimen are compared with the DIC strain fields and the FE simulation of a sharp crack. The simulations and the experiments are conducted at a nominal stretch rate of 0.02/s. These comparisons are plotted in Figure S3 (DIC: circle; sharp crack FEM: dashed line; notched crack FEM: solid line) at a nominal stretch of 1.5. It is clear that  and  agree better with the FE simulation of the blunted crack.

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| Figure S2. The geometry of the sample used in DIC tests. Figure taken from [33]. |

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| (a) | (b) |
| Figure S3. Comparisons of the Lagrangian strain fields of the single edge crack sample. The circles are DIC measurements, the dashed lines are FE results with a perfectly sharp crack, and the solid lines are FE results of a finite size crack with geometry shown in Figure S2. | |

**4. Comparison with DIC: full field comparison through contour plots**

In Figure 8 and Figure S3 we compared the Lagrangian strains directly ahead of the crack tip (blue dashed line marked in Figure S4a below), which is the symmetry axis of the specimen. Here we compare the strain field of the full sample to further prove that our model captures the full field behavior of our PVA gel, and to demonstrate our ability to carry out FEA and DIC measurements. The nominal strain is defined by

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where  is the left stretch tensor. The contour plots of  obtained from FEM and DIC are shown in Figure S5. Both contours are plotted on the same color scale and in the same region of interest. The crack face is marked white in the contour plot from FEM in Figure S5a. It can be seen that DIC measurements agree very well with the FEM simulation, throughout the whole field.

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| (a) | (b) |
| Figure S4. (a) Schematics drawing of the crack sample we use in our DIC tests. The crack is on the right edge, and we use the spatial coordinates drawn in red in the following comparisons (Figures S4b, S5a, S5b, S6). The origin is placed at the crack tip.  (b) DIC image at the reference frame, with the region of interest highlighted in yellow. | |

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| (a) | (b) |
| Figure S5. Contour plots of . The origin is at the crack tip, and the coordinate system is shown in Figure S4a. The color bars and the region of interest are the same in the two contour plots. | |

**5. Comparison with DIC: displacement along a perpendicular path**

In our DIC analysis, we used a relatively small strain radius of 2, which avoided the over-smoothing due to sampling too many displacement points for strain calculation, so the strains given by DIC are valid. Nonetheless, we can look at the displacement measurements to eliminate possible effect of smoothing. In Figure S6 we plot the vertical displacement along a path perpendicular to the crack face from both FEM and DIC. The path is the green dashed line drawn in the schematics in Figure S4a. The FEM and DIC results agree very well.

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| Figure S6. Comparison of the vertical displacement  along a path perpendicular to the crack face. |

**6. Comparison between 3D FEM and plane stress FEM**

To validate the plane stress assumption, we compare the plane stress FEM with a 3D FEM of the single edge crack sample shown in Figures 6a-c. The 3D FEM was reported in our previous paper32. Crack opening displacements  and the dominant true stress component  directly in front of the crack tip are extracted at three different applied nominal stretch ratios , ,  and these 2D and 3D FEM results are compared in Figure S7 below.

The crack opening displacements between plane stress and 3D are almost indistinguishable. The crack opening displacements from the plane stress simulations are slightly higher than the 3D simulations only very close to the crack tip, and these discrepancies are sufficiently small so they are not reflected in our comparison with experiments.

The singularity of the dominant stresses  obtained from 3D FEM is slightly smaller than those from the plane stress FEM. This lesser singularity could be due to both the 3D effect and the coarse mesh in the 3D FEM. Indeed, if we increase the mesh size in our plane stress calculation, we found that the stress field is consistent with the 3D results. This support our claim that the region of dominance of the  field is sufficiently small so a finer mesh is needed for the 3D FEM.

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| Figure S7a. Comparisons of crack opening displacements between plane stress FEM and 3D FEM at three different nominal stretch ratios , ,  of the crack sample shown in Figures 6a-c. The sample is loaded at constant nominal stretch rate  until failure (at ). |

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| Figure S7b. Comparisons of dominant true stress between plane stress FEM and 3D FEM at a nominal stretch ratio  of the crack sample shown in Figures 6a-c. The sample is loaded at constant nominal stretch rate  until failure (at ). |