Spatiotemporal Integration in Plant Tropisms
SUPPLEMENTARY MATERIAL

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I. TIME AND SPACE INTEGRATION

We note that $\frac{\partial\theta(s,t)}{\partial s} \equiv \kappa(s,t)$ is the local curvature, and we can rewrite Eq. 2

$$\frac{\partial\kappa(s,t)}{\partial t} = -\gamma\kappa(s,t) - \int_{-\infty}^{t} \beta(\tau)\mu(t-\tau)\sin(\theta(s,\tau)-\theta_p)d\tau. \tag{1}$$

We use a simple explicit Euler scheme to integrate Eq. 1 over time, for each point s along the organ. The organ is set to size $L = 1$, and is divided into $NL = 100$ bins, i.e. each segment size is $\mathrm{d}s = L/NL = 0.01$. The time step is fixed at $\mathrm{d}t = 0.005$. Within the Euler scheme, we generally have $\kappa(s,t+1) = \kappa(s,t) + \mathrm{d}t \cdot \frac{\partial\kappa(s,t)}{\partial t}$, where we substitute the derivative with the right-hand-side of Eq. 1, yielding:

$$\kappa(t+1,s) = \kappa(t,s) + \mathrm{d}t \cdot (-\gamma\kappa(t,s) - \sum_{n=0}^{N} \mathrm{d}t\beta(t-n)\sin(\theta(s,n) - \frac{\pi}{2})e^{(-n\mathrm{d}t/\tau_c)}. \tag{2}$$

Here we demonstrated the case of an exponential response function. After each time step we also integrate the derived curvature over space, in order to get the local angle, since $\theta(s,t) = \theta(s=0,t) + \int_{s'=0}^{s}\kappa(s',t)$. We therefore loop over the organ segments $s = 0,..,NL$:

$$\theta(s,t) = \theta(s-1,t) + \mathrm{d}s \cdot \kappa(s,t) \tag{3}$$

II. ANALYTIC CALCULATION OF THE RESPONSE FUNCTION

In order to extract the form of the kernel response function from the kinematics of tropic responses, we solve Eq. 2 for the case of a pulse stimulus, by substituting $\beta(t) = \beta_0\delta(t_0 = 0)$ in the linearized limit (i.e we substitute $\sin(\theta(s,t) - \theta_p)$ with $(\theta(s,t) - \theta_p)$. Since we know that $\int dt f(t-\tau)\delta(t) = f(\tau)$, substituting a pulse stimulus eliminates the convolution, and together with the initial condition $\theta(s, t = 0) = 0$ leads to:

$$\frac{\partial^2\theta(s,t)}{\partial t\partial s} + \gamma\frac{\partial\theta(s,t)}{\partial s} = -\beta_0\theta_p\mu(t). \tag{4}$$

We now recall that $\int_0^L ds\frac{df(s)}{ds} = f(L) - f(0)$. Therefore if we integrate over Eq. 4 we get the following:

$$\frac{\partial\theta(L,t)}{\partial t} - \frac{\partial\theta(0,t)}{\partial t} + \gamma\theta(L,t) - \gamma\theta(0,t) = -\beta_0\theta_p\mu(t)\int_0^L ds. \tag{5}$$

We substitute the clamped boundary conditions, $\theta(s = 0,t) = \frac{\partial\theta(0,t)}{\partial t} = 0$, and $\int_0^L ds = L$, and rearrange the equation, finally leading to Eq. 4:

$$\mu(t) = \frac{1}{L\theta_p\beta_0}\left(\frac{\partial\theta(L,t)}{\partial t} + \gamma\theta(L,t)\right). \tag{6}$$

Moreover, rescaling the characteristic time and length scales in the problem using $l = L/L_c$, $\tau = t/T_c$ and $\varphi = \theta(L,t)/\theta_p$, we can write Eq. 4 yielding:

$$\mu(\tau) = \frac{1}{l}\left(\frac{\partial\varphi(l,\tau)}{\partial\tau} + \varphi(l,\tau)\right), \tag{7}$$

where $L_c = \gamma/\beta$ is the convergence length, and is given by the decay length of the exponential toward the vertical, and it results from the balance between graviception and proprioception [20].