# Supplementary Material for

# ‘Electromechanical vibration of microtubules and its application in biosensors’

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## Details of dynamic analysis for MSM model: Structure under loads that vary sinusoidally at the known frequency

Consider the general equation of motion for a structural system

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| --- | --- |
| $$M\ddot{u}+C\dot{u}+Ku=F$$ | (S1) |

where $\ddot{u}$ denotes the acceleration vector, $\dot{u}$ denotes the velocity vector, $u$ denotes the nodal displacement vector, $F$ denotes the applied force vector, $M$ denotes the structural mass matrix, $C$ denotes the structural damping matrix, $K$ denotes the structural stiffness matrix.

As stated above, all points in the structure are moving at the same known frequency, however, not necessarily in phase. Also, it is known that the presence of damping causes phase shifts. Therefore, the displacements may be defined as

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| $$u=\{u\_{max}e^{iφ}\}e^{iΩt}$$ | (S2) |

where $u\_{max}$ denotes the maximum displacement, $i$ denotes square root of -1, $Ω$ denotes imposed circular frequency (radians/time) = 2π*f*, $f$ denotes the imposed frequency (cycles/time), $t$ denotes time, $φ$ denotes the displacement phase shift (radians).

The above equation can be written as $u=\{u\_{max}(cosφ+isinφ)\}e^{iΩt}$ or as $u=(\{u\_{1}\}+i\{u\_{2}\})e^{iΩt}$.

where $\{u\_{1}\}=\{u\_{max}cosφ\}$ is the real displacement vector and $\{u\_{2}\}=\{u\_{max}sinφ\}$ is the imaginary displacement vector.

The force vector can be specified analogously to the displacement:

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| $$F=\{F\_{max}e^{iψ}\}e^{iΩt}$$ | (S3) |
| $$F=\{F\_{max}(cosψ+isinψ)\}e^{iΩt}$$ | (S4) |
| $$F=(\{F\_{1}\}+i\{F\_{2}\})e^{iΩt}$$ | (S5) |

where $F\_{max}$ denotes force amplitude, $ψ$ denotes force phase shift (radians), $\{F\_{1}\}=\{F\_{max}cosψ\}$ is the real force vector, $\{F\_{2}\}=\{F\_{max}sinψ\}$ is the imaginary force vector$ $.

Then we could have

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| $$(-Ω^{2}M+iΩC+K)(\{u\_{1}\}+i\{u\_{2}\})e^{iΩt}=(\{F\_{1}\}+i\{F\_{2}\})e^{iΩt}$$ | (S6) |

The dependence on time $(e^{iΩt})$ is the same on both sides of the equation and may therefore be removed and we have

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| $$(K-Ω^{2}M+iΩC)(\{u\_{1}\}+i\{u\_{2}\})=(\{F\_{1}\}+i\{F\_{2}\})$$ | (S7) |

Solving this equation can help us obtain the displacement of the structure under this dynamic situation (periodic sinusoidal force).

In the present paper, the Eigenvalue vibration analysis was also performed as a comparison and the details of the method for the Eigenvalue vibration analysis could be found in [[1](#_ENREF_1)].

In this study, the damping effect of the surrounding medium was considered through viscous damping forces on monomer which was described in Sec. 2 of the Supplementary Material. Therefore, the structural damping matrix $C$ was ignored. The method to obtain the structural mass matrix $M$ and the structural stiffness matrix $K$ is introduced here.

 In the present MSM model, the mass for beam element was calculated as $M\_{t}=ρAl$, where $ρ$ is the density of the beam element and *l* is the length of the beam.

 Then the element mass matrix $M^{e}$is calculated as per finite element method and is given by:

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| --- | --- |
| $$M^{e}=\frac{M\_{t}}{2}\left[\begin{matrix}1&0&0&0&0&0&0&0&0&0&0&0\\0&1&0&0&0&0&0&0&0&0&0&0\\0&0&1&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&1&0&0&0&0&0\\0&0&0&0&0&0&0&1&0&0&0&0\\0&0&0&0&0&0&0&0&1&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\\0&0&0&0&0&0&0&0&0&0&0&0\end{matrix}\right]$$ | (S8) |

The element elastic stiffness matrix $K^{e}$ of the MSM model is calculated as:

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| $$K^{e}=\left[\begin{matrix}K\_{aa}&K\_{ab}\\K\_{ab}^{T}&K\_{bb}\end{matrix}\right]$$ | (S9) |

where the sub-matrices are obtained via the following equations and the beam stiffnesses *YA*, *YI*, *SJ* given by Eqs. 5 in the main text.

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| $$K\_{aa}=\left[\begin{matrix}YA/l&0&0&0&0&0\\0&12YI/l^{3}&0&0&0&6YI/l^{2}\\0&0&12YI/l^{3}&0&-6YI/l^{2}&0\\0&0&0&SJ/l&0&0\\0&0&-6YI/l^{2}&0&4YI/l&0\\0&6YI/l^{2}&0&0&0&4YI/l\end{matrix}\right]$$ | (S10) |
| $$K\_{ab}=\left[\begin{matrix}-YA/l&0&0&0&0&0\\0&-12YI/l^{3}&0&0&0&6YI/l^{2}\\0&0&-12YI/l^{3}&0&-6YI/l^{2}&0\\0&0&0&-SJ/l&0&0\\0&0&6YI/l^{2}&0&2YI/l&0\\0&-6YI/l^{2}&0&0&0&2YI/l\end{matrix}\right]$$ | (S11) |
| $$K\_{bb}=\left[\begin{matrix}YA/l&0&0&0&0&0\\0&12YI/l^{3}&0&0&0&-6YI/l^{2}\\0&0&12YI/l^{3}&0&6YI/l^{2}&0\\0&0&0&SJ/l&0&0\\0&0&6YI/l^{2}&0&4YI/l&0\\0&-6YI/l^{2}&0&0&0&4YI/l\end{matrix}\right]$$ | (S12) |

 The assembling procedure from the elemental matrices ($M^{e}$, $K^{e}$) to the structural mass matrix $M$ and the structural stiffness matrix $K$ follows the node-related technique in the finite element method [[2](#_ENREF_2)].

## A brief introduction to the damping model of MT vibration in cytosol

 The slide film damping theory was utilized here to characterize the energy dissipation due to the viscous flow in cytosol during the high-frequency vibration of MTs. The damping effect caused by microfluid between two moving microscale objects can be effectively represented by this theory [[3](#_ENREF_3), [4](#_ENREF_4)]. As can be seen from the experimental observation of a cell, the MTs in cytosol are surrounded by microscale objects [[5](#_ENREF_5)]. During the high-frequency vibration, the relative motion between the MT and the surfaces will also lead to the energy dissipation in cytosol and the damping of the vibration. Here, Couette fluid model [[3](#_ENREF_3)] was employed for the cytosol and the governing equation obtained for the sinusoidal motion is as follows [[4](#_ENREF_4)]:

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| $$ρ\frac{∂v}{∂t}=η\frac{∂^{2}v}{∂z^{2}}$$ |  (S13) |

where *v* is the velocity, *ρ* is the density of fluid and *η* is the dynamic viscosity of the fluid, *z* is the distance between a point in fluid and the moving solid surface.

 In the theory, the transition from Couette to Stokes flow occurs at a cut-off frequency $f\_{c}=η/2πρh^{2}$, where the *h* is the spacing between the MT and adjacent filaments and set to 20 nm here [[6](#_ENREF_6)]). Thus, 277MHz was obtained as the cut-off frequency which is far beyond the scope (1-50 MHz) considered and shows that the Couette flow model is suitable for the cytosol around the vibrating MT. Eqs. S13 gives the velocity profile in the fluid field and the damping force is $F\_{d}=ηAv\_{t}/h$ where $v\_{t}$ is the velocity of tubulin and *A* is the tubulin-water contact area. Here the *η* takes 0.000697 Pa. s [[7](#_ENREF_7)] and *A ≈ 23.5 nm2* was measured with atomic structure of monomeric tubulin labelled by PDB ID code 1TUB [[8](#_ENREF_8)].

 In this study, two types of MT-cytosol interfaces were assumed and compared with each other in terms of damping effect, i.e., (1) the non-slip boundary condition associated with a continuous MT-cytosol interface where *uliquid* = *usolid* [[9](#_ENREF_9)] and the damping forces *Fd*. (2) the slip-boundary condition at the interface where *uliquid* ≠ *usolid* and the damping force is *Fd × P* on the monomers. Here, coefficient *P* ranges from 0.0001 to 0.1, reflecting different reduced damping effect.

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