Supplementary material for " Patterned Surface Charges coupled with Thermal Gradients may Create Giant Augmentations of Solute Dispersion in Electro-Osmosis of Viscoelastic Fluids"

A: Analytical Solution Procedure

Using the non-dimensionalisation scheme discussed earlier, the continuity, momentum, energy and charge distribution equations are rewritten in the following way:

$$\text{Continuity}: \quad \frac{\partial \bar{u}}{\partial \bar{x}}+\frac{\partial \bar{v}}{\partial \bar{y}}=0 \tag{A1}$$

$$\left.\begin{aligned}
&x\text{-momentum}: \quad 0=-\frac{\partial \bar{p}}{\partial \bar{x}}+\chi\frac{\partial}{\partial \bar{x}}\left\{\exp\left(-\xi\bar{T}\right)\bar{\tau}_{xx}\right\}+\frac{\partial}{\partial \bar{y}}\left\{\exp\left(-\xi\bar{T}\right)\bar{\tau}_{yx}\right\}-\frac{\bar{\kappa}_o^2\,\bar{\psi}}{\xi\,\beta_1\bar{T}+1}\frac{d\bar{\phi}}{d\bar{x}}\\
&\qquad -\bar{\kappa}_o^2\,\lambda\left\{\frac{\bar{\psi}}{\xi\,\beta_1\bar{T}+1}-\bar{\psi}\right\}\frac{\partial\bar{\psi}}{\partial\bar{x}}-\frac{\xi\,\beta_2}{2}\left\{\frac{\chi^2}{\lambda}\left(\frac{d\bar{\phi}}{d\bar{x}}+\lambda\frac{\partial\bar{\psi}}{\partial\bar{x}}\right)^2+\lambda\left(\frac{\partial\bar{\psi}}{\partial\bar{y}}\right)^2\right\}\frac{\partial\bar{T}}{\partial\bar{x}}\\
&y-\text{momentum}: \quad 0=-\frac{\partial \bar{p}}{\partial \bar{y}}+\chi^2\frac{\partial}{\partial \bar{x}}\left\{\exp\left(-\xi\bar{T}\right)\bar{\tau}_{xy}\right\}+\chi\frac{\partial}{\partial \bar{y}}\left\{\exp\left(-\xi\bar{T}\right)\bar{\tau}_{yy}\right\}\\
&\qquad -\bar{\kappa}_o^2\lambda\left\{\left(\frac{\bar{\psi}}{\xi\,\beta_1\bar{T}+1}\right)-\bar{\psi}\right\}\frac{\partial\bar{\psi}}{\partial\bar{y}}-\frac{\xi\,\beta_2}{2}\left\{\frac{\chi^2}{\lambda}\left(\frac{d\bar{\phi}}{d\bar{x}}+\lambda\frac{\partial\bar{\psi}}{\partial\bar{x}}\right)^2+\lambda\left(\frac{\partial\bar{\psi}}{\partial\bar{y}}\right)^2\right\}\frac{\partial\bar{T}}{\partial\bar{y}}
\end{aligned}\right\} \tag{A2}$$

$$\begin{aligned}
&\text{Energy Equation}: \; Pe_T\left(\bar{u}\frac{\partial\bar{T}}{\partial\bar{x}}+v\frac{\partial\bar{T}}{\partial\bar{y}}\right)=\chi\frac{\partial}{\partial\bar{x}}\left\{\left(1+\xi\,\beta_3\bar{T}\right)\frac{\partial\bar{T}}{\partial\bar{x}}\right\}+\frac{1}{\chi}\frac{\partial}{\partial\bar{y}}\left\{\left(1+\xi\,\beta_3\bar{T}\right)\frac{\partial\bar{T}}{\partial\bar{y}}\right\}\\
&\qquad +\left(1+\xi\,\beta_4\bar{T}\right)\left\{\left(\frac{d\bar{\phi}}{d\bar{x}}\right)^2+2\lambda\frac{d\bar{\phi}}{d\bar{x}}\frac{\partial\bar{\psi}}{\partial\bar{x}}+\lambda^2\left(\frac{\partial\bar{\psi}}{\partial\bar{x}}\right)^2\right\}+\frac{\lambda^2\left(1+\xi\,\beta_4\bar{T}\right)}{\chi^2}\left(\frac{\partial\bar{\psi}}{\partial\bar{y}}\right)^2
\end{aligned} \tag{A3}$$

In equations (A2)-(A3), χ is the ratio of the length scales in the longitudinal and transverse directions $\left(\chi=h/l\right)$ and Pe_T is the thermal Peclet number $\left(Pe_T=\rho\,C_p\,u_{HS}\,h/k_{ref}\right)$. Here, $\lambda=\zeta_{ref}/\phi_{ref}$ is the ratio of the induced potential and the applied potential and $\beta_1=1/\alpha_6\,T_{ref}$, $\beta_2=\alpha_5/\alpha_6$, $\beta_3=\alpha_3/\alpha_6$, $\beta_4=\alpha_4/\alpha_6$ are the parameters showing the temperature dependence of the physical properties while $\bar{\kappa}_0=\kappa_0\,h$ is the inverse of the dimensionless EDL thickness with $\kappa_0=\sqrt{2n_0\,z^2\,e^2/\left(\varepsilon_{ref}k_B\,T\right)}$. Besides, the thermal perturbation to the system can be characterized by the parameter $\xi=\alpha_6\,\Delta T$. Also, we define a new variable $\nu=\left(h_T\,h/k_{ref}\right)$ to take into account the convective heat loss to the surrounding. Interestingly, this variable is nothing but the well-known dimensionless number Biot number (Bi) which is the ratio of the conductive heat transfer from the solid surface to the convective heat transfer in the surrounding. Now, considering the case of low surface potential, the simplified charge distribution along with the current continuity equations are presented below

$$\left.\begin{aligned}&\text{Charge Distribution}: \ \frac{\bar{\kappa}_0^2\,\bar{\psi}}{\xi\,\beta_1\,\bar{T}+1}=\frac{\chi^2}{\lambda}\frac{\partial}{\partial\bar{x}}\left\{\left(1-\xi\,\beta_2\,\bar{T}\right)\left(\frac{d\bar{\phi}}{d\bar{x}}+\lambda\frac{\partial\bar{\psi}}{\partial\bar{x}}\right)\right\}+\frac{\partial}{\partial\bar{y}}\left\{\left(1-\xi\,\beta_2\,\bar{T}\right)\frac{\partial\bar{\psi}}{\partial\bar{y}}\right\}\\&\text{Current Continuity}: \ \int_{-1}^{1}\frac{\partial}{\partial\bar{x}}\left\{\left(1+\xi\,\beta_4\,\bar{T}\right)\left(\frac{d\bar{\phi}}{d\bar{x}}+\lambda\frac{\partial\bar{\psi}}{\partial\bar{x}}\right)\right\}d\,\bar{y}=0\end{aligned}\right\}\quad\text{(A4)}$$

After using the relevant scales, the dimensionless forms of the stress components for a viscoelastic fluid take the following form:

$$\left.\begin{aligned}&2\chi\frac{\partial\bar{u}}{\partial\bar{x}}=\left\{1+\frac{\delta De_{\bar{\kappa}_o}}{\bar{\kappa}_0}\exp\left(-\xi\,\bar{T}\right)\left(\bar{\tau}_{xx}+\bar{\tau}_{yy}\right)\right\}\bar{\tau}_{xx}+\frac{De_{\bar{\kappa}_o}\chi}{\bar{\kappa}_0}\left\{\begin{aligned}&\bar{u}\frac{\partial}{\partial\bar{x}}\left\{\exp\left(-\xi\,\bar{T}\right)\bar{\tau}_{xx}\right\}+\bar{v}\frac{\partial}{\partial\bar{y}}\left\{\exp\left(-\xi\,\bar{T}\right)\bar{\tau}_{xx}\right\}\\&-2\exp\left(-\xi\,\bar{T}\right)\frac{\partial\bar{u}}{\partial\bar{x}}\bar{\tau}_{xx}-\frac{2}{\chi}\exp\left(-\xi\,\bar{T}\right)\frac{\partial\bar{u}}{\partial\bar{y}}\bar{\tau}_{yx}\end{aligned}\right\}\\&2\chi\frac{\partial\bar{v}}{\partial\bar{y}}=\left\{1+\frac{\delta De_{\bar{\kappa}_o}}{\bar{\kappa}_0}\exp\left(-\xi\,\bar{T}\right)\left(\bar{\tau}_{xx}+\bar{\tau}_{yy}\right)\right\}\bar{\tau}_{yy}+\frac{De_{\bar{\kappa}_o}\chi}{\bar{\kappa}_0}\left\{\begin{aligned}&\bar{u}\frac{\partial}{\partial\bar{x}}\left\{\exp\left(-\xi\,\bar{T}\right)\bar{\tau}_{yy}\right\}+\bar{v}\frac{\partial}{\partial\bar{y}}\left\{\exp\left(-\xi\,\bar{T}\right)\bar{\tau}_{yy}\right\}\\&-2\chi\exp\left(-\xi\,\bar{T}\right)\frac{\partial\bar{v}}{\partial\bar{x}}\bar{\tau}_{xy}-2\exp\left(-\xi\,\bar{T}\right)\frac{\partial\bar{v}}{\partial\bar{y}}\bar{\tau}_{yy}\end{aligned}\right\}\\&\frac{\partial\bar{u}}{\partial\bar{y}}+\chi^2\frac{\partial\bar{v}}{\partial\bar{x}}=\left\{1+\frac{\delta De_{\bar{\kappa}_o}}{\bar{\kappa}_0}\exp\left(-\xi\,\bar{T}\right)\left(\bar{\tau}_{xx}+\bar{\tau}_{yy}\right)\right\}\bar{\tau}_{xy}+\frac{De_{\bar{\kappa}_o}\chi}{\bar{\kappa}_0}\left\{\begin{aligned}&\bar{u}\frac{\partial}{\partial\bar{x}}\left\{\exp\left(-\xi\bar{T}\right)\bar{\tau}_{xy}\right\}+\bar{v}\frac{\partial}{\partial\bar{y}}\left\{\exp\left(-\xi\bar{T}\right)\bar{\tau}_{xy}\right\}\\&-\chi\exp\left(-\xi\bar{T}\right)\frac{\partial\bar{v}}{\partial\bar{x}}\bar{\tau}_{xx}-\frac{\exp\left(-\xi\bar{T}\right)}{\chi}\frac{\partial\bar{u}}{\partial\bar{y}}\bar{\tau}_{yy}\end{aligned}\right\}\end{aligned}\right\}\quad\text{(A5)}$$

where the Deborah number $\left(De_{\bar{\kappa}_0}=\lambda_{ref}\,\kappa_0\,u_{HS}\right)$ represents the extent of viscoelasticity of the fluid wherein $De_{\bar{\kappa}_0}=0$ corresponds to Newtonian fluid. Now, the boundary conditions described by equation (8) of the manuscript are rewritten in their respective non-dimensional forms

$$\left.\begin{aligned}&\bar{u}\left(\bar{y}=\pm1\right)=0;\ \ \bar{v}\left(\bar{y}=\pm1\right)=0;\ \ \bar{p}\left(\bar{x}=0\right)=0;\ \ \bar{p}\left(\bar{x}=1\right)=0;\\&\bar{T}\left(x=0\right)=0;\ \ \bar{T}\left(\bar{x}=1\right)=0;\ \ \bar{\phi}\left(x=0\right)=1;\ \ \bar{\phi}\left(\bar{x}=1\right)=0;\\&\left(1+\xi\,\beta_3\,\bar{T}\right)\left(\partial\bar{T}/\partial\bar{y}\right)\Big|_{\bar{y}=\pm1}=\mp\nu\,\bar{T};\ \ \left(\partial\bar{T}/\partial\bar{y}\right)\Big|_{\bar{y}=0}=0;\\&\qquad\bar{\psi}\left(\bar{y}=\pm1\right)=\alpha_1+\alpha_2\cos\left(\omega\,\bar{x}\right)\end{aligned}\right\}\quad\text{(A6)}$$

where $\omega=\omega_t\,l$ is the patterning frequency of modulation in dimensionless form. To obtain the flow and temperature fields from the set of above dimensionless forms, we have performed an asymptotic approach followed by the classical lubrication approximation theory. [1–4]

In typical microfluidic applications, the length scale in the transverse coordinate is very small as compared to the longitudinal coordinates $\left(l\gg h\right)$. In the limit of $\chi\to0$, the terms involving $\mathrm{O}\left(\chi\right)$ and $\mathrm{O}\left(\chi^2\right)$ can be discarded and the simplified momentum and the stress components become

$$
\left.
\begin{aligned}
\frac{\partial \bar{p}}{\partial \bar{x}} &= \frac{\partial}{\partial \bar{y}}\left\{\exp\left(-\xi \bar{T}\right)\bar{\tau}_{yx}\right\} - \frac{\bar{\kappa}_o^2 \bar{\psi}}{\xi \beta_1 \bar{T}+1}\frac{d\bar{\phi}}{d\bar{x}} - \bar{\kappa}_o^2 \lambda \left\{\frac{\bar{\psi}}{\xi \beta_1 \bar{T}+1} - \bar{\psi}\right\}\frac{\partial \bar{\psi}}{\partial \bar{x}} \\
&\quad - \frac{\xi \beta_2}{2}\left\{\frac{\chi^2}{\lambda}\left(\frac{d\bar{\phi}}{d\bar{x}} + \lambda \frac{\partial \bar{\psi}}{\partial \bar{x}}\right)^2 + \lambda \left(\frac{\partial \bar{\psi}}{\partial \bar{y}}\right)^2\right\}\frac{\partial \bar{T}}{\partial \bar{x}} \\
\frac{\partial \bar{p}}{\partial \bar{y}} &= -\bar{\kappa}_o^2 \lambda \left\{\frac{\bar{\psi}}{\xi \beta_1 \bar{T}+1} - \bar{\psi}\right\}\frac{\partial \bar{\psi}}{\partial \bar{y}} - \frac{\xi \beta_2}{2}\left\{\frac{\chi^2}{\lambda}\left(\frac{d\bar{\phi}}{d\bar{x}} + \lambda \frac{\partial \bar{\psi}}{\partial \bar{x}}\right)^2 + \lambda \left(\frac{\partial \bar{\psi}}{\partial \bar{y}}\right)^2\right\}\frac{\partial \bar{T}}{\partial \bar{y}}
\end{aligned}
\right\} \quad \text{(A7)}
$$

$$
\left.
\begin{aligned}
\left\{1 + \frac{\delta De_{\bar{\kappa}_o}}{\bar{\kappa}_0}\exp\left(-\xi \bar{T}\right)\left(\bar{\tau}_{xx} + \bar{\tau}_{yy}\right)\right\}\bar{\tau}_{xx} &= \frac{2 De_{\bar{\kappa}_o}}{\bar{\kappa}_0}\exp\left(-\xi \bar{T}\right)\frac{\partial \bar{u}}{\partial \bar{y}}\bar{\tau}_{yx} \\
\left\{1 + \frac{\delta De_{\bar{\kappa}_o}}{\bar{\kappa}_0}\exp\left(-\xi \bar{T}\right)\left(\bar{\tau}_{xx} + \bar{\tau}_{yy}\right)\right\}\bar{\tau}_{yy} &= 0 \\
\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{De_{\bar{\kappa}_o}}{\bar{\kappa}_0}\exp\left(-\xi \bar{T}\right)\frac{\partial \bar{u}}{\partial \bar{y}}\bar{\tau}_{yy} &= \left\{1 + \frac{\delta De_{\bar{\kappa}_o}}{\bar{\kappa}_0}\exp\left(-\xi \bar{T}\right)\left(\bar{\tau}_{xx} + \bar{\tau}_{yy}\right)\right\}\bar{\tau}_{xy}
\end{aligned}
\right\} \quad \text{(A8)}
$$

From equation (A8), it is clear that $\bar{\tau}_{yy} = 0$ and a relationship between the stress components $\bar{\tau}_{xx}$ and $\bar{\tau}_{yx}$ can be established. Meanwhile in the energy equation, the conduction terms cannot be neglected even in the limit of $\chi \to 0$ because of their relative strengths with respect to the other terms. This can be done simply by doing an order of magnitude analysis of these two terms where one can compare their relative strengths with respective to heat generation term due to Joule heating. The characteristic temperature in the axial and transverse directions are scaled as $\Delta T_x \sim \sigma_{ref} E_{ref}{}^2 l^2 / k_{ref}$ and $\Delta T_y \sim \sigma_{ref} E_{ref}{}^2 h^2 / k_{ref}$ respectively and hence, $\Delta T_x / \Delta T_y \sim l^2/h^2 \sim \chi^{-2} \gg 1$. Since the scales of the diffusive terms are expressed as $\chi \Delta T_x$ and $\Delta T_y / \chi$ respectively, they are comparable to each other (i.e. $\chi \Delta T_x$ ~ $\Delta T_y / \chi$) and also in the same order with the heat generation term and therefore, these terms cannot be neglected in the energy equation even in the limit of $\chi \to 0$. [3] Similarly, to determine the relative contributions of the convective components, we use the scales of ΔT_x and ΔT_y and their ratio becomes $\Delta T_x / \Delta T_y \sim l^2/h^2 \gg 1$, *i.e.*, $\partial \bar{T} / \partial \bar{y} \ll \partial \bar{T} / \partial \bar{x}$. Additionally, one can assume that the surface potential is very small as compared to the applied potential, i.e., $\zeta_{ref} / \phi_{ref} = \lambda \ll 1$ and the terms involving $\mathrm{O}(\lambda)$ and its higher orders can be neglected thereby resulting the following simplified forms:

$$
\left.
\begin{aligned}
\frac{\partial \bar{p}}{\partial \bar{x}} &= \frac{\partial}{\partial \bar{y}}\left\{\exp\left(-\xi \bar{T}\right)\bar{\tau}_{yx}\right\} - \frac{\bar{\kappa}_o^2 \bar{\psi}}{\xi \beta_1 \bar{T}+1}\frac{d\bar{\phi}}{d\bar{x}} - \frac{\xi \beta_2 \chi^2}{2\lambda}\left(\frac{d\bar{\phi}}{d\bar{x}}\right)^2 \frac{\partial \bar{T}}{\partial \bar{x}} \\
\frac{\partial \bar{p}}{\partial \bar{y}} &= -\frac{\xi \beta_2 \chi^2}{2\lambda}\left(\frac{d\bar{\phi}}{d\bar{x}}\right)^2 \frac{\partial \bar{T}}{\partial \bar{y}}
\end{aligned}
\right\} \quad \text{(A9)}
$$

$$
Pe_T\left(\bar{u}\frac{\partial \bar{T}}{\partial \bar{x}}\right) = \chi \frac{\partial}{\partial \bar{x}}\left\{\left(1 + \xi \beta_3 \bar{T}\right)\frac{\partial \bar{T}}{\partial \bar{x}}\right\} + \frac{1}{\chi}\frac{\partial}{\partial \bar{y}}\left\{\left(1 + \xi \beta_3 \bar{T}\right)\frac{\partial \bar{T}}{\partial \bar{y}}\right\} + \left(1 + \xi \beta_4 \bar{T}\right)\left(\frac{d\bar{\phi}}{d\bar{x}}\right)^2 \quad \text{(A10)}
$$

Now we compare the relative contributions of the terms of the charge distribution described by equation (3) of the manuscript. Choosing appropriate scales of the respective parameters, i.e. $\varepsilon \sim \varepsilon_{ref}$, $\phi \sim \phi_{ref}$, $\psi \sim \zeta_{ref}$, $x \sim l$, and $y \sim \lambda_D$, the first term on the left hand side of equation (3b) becomes $\sim \varepsilon_{ref}\left(\phi_{ref}+\zeta_{ref}\right)/l^2$ while the second term is scaled as $\sim \varepsilon_{ref}\,\zeta_{ref}/\lambda_D{}^2$. Hence, the ratio becomes $\sim \frac{\varepsilon_{ref}}{l^2}\left(\phi_{ref}+\zeta_{ref}\right)\Big/\frac{\varepsilon_{ref}\,\zeta_{ref}}{\lambda_D{}^2} \sim \frac{\lambda_D{}^2}{l^2}+\frac{\lambda_D{}^2}{l^2\lambda} \sim \frac{\lambda_D{}^2}{l^2}+\frac{\chi^2}{\bar{\kappa}_0^2\lambda}$ where $\lambda_D << l$ and $\frac{\chi^2}{\bar{\kappa}_0^2\lambda} << 1$; which is negligible compared to the second term and the simplified form is now written below

$$\left.\begin{aligned} &\frac{\bar{\kappa}_0^2\,\bar{\psi}}{\xi\,\beta_1\,\bar{T}+1}=\frac{\partial}{\partial\bar{y}}\left\{\left(1-\xi\,\beta_2\,\bar{T}\right)\frac{\partial\bar{\psi}}{\partial\bar{y}}\right\} \\ &\text{and} \qquad \int_{-1}^{1}\frac{\partial}{\partial\bar{x}}\left\{\left(1+\xi\,\beta_4\,\bar{T}\right)\left(\frac{d\bar{\phi}}{d\bar{x}}\right)\right\}d\bar{y}=0 \end{aligned}\right\} \tag{A11}$$

By choosing the typical values of the pertinent parameters (these values are shown in the Results and Discussion section), one can show that $\frac{\xi\,\beta_2\,\chi^2}{2\,\lambda} \ll 1$ and the momentum components are then reduced to the following form

$$\left.\frac{d\bar{p}}{d\bar{x}}=\frac{\partial}{\partial\bar{y}}\left\{\exp\left(-\xi\bar{T}\right)\bar{\tau}_{yx}\right\}-\frac{\bar{\kappa}_o^2\,\bar{\psi}}{\xi\,\beta_1\,\bar{T}+1}\frac{d\bar{\phi}}{d\bar{x}} \qquad \text{and} \qquad \frac{\partial\bar{p}}{\partial\bar{y}}=0\ \right\} \tag{A12}$$

Since the axial variation of the temperature in the *x* co-ordinate is more significant compared to the *y* co-ordinate, one can expand the temperature distribution in an asymptotic series in the following manner

$$\bar{T}=\bar{T}_0\left(\bar{x}\right)+\nu\,\bar{T}_1\left(\bar{x},\bar{y}\right)+\nu^2\,\bar{T}_2\left(\bar{x},\bar{y}\right)+O\left(\nu^3\right) \tag{A13}$$

where ν characterizes the rate of heat loss to the surrounding. Now, we utilize this expansion along with the two thermal boundary conditions and integrate the energy equation over the entire domain

$$\frac{1}{2}Pe\frac{d\bar{T}_0}{d\bar{x}}\left\{\int_{-1}^{1}\bar{u}\,d\,\bar{y}\right\}=\chi\frac{d^2\bar{T}_0}{d\bar{x}^2}-\frac{\nu\,\bar{T}_0}{\chi}+\left(1+\xi\,\beta_4\,\bar{T}_0\right)\left(\frac{d\bar{\phi}}{d\bar{x}}\right)^2 \tag{A14}$$

As discussed earlier in the manuscript, ν turns out to be a small quantity as compared to unity. Physically, lower value of ν corresponds to lesser convective heat loss to the surrounding which in turn influences slightly the velocity and temperature distribution within the flow domain. To incorporate this small change, we have chosen ν as perturbation parameter along with ξ. Subsequently, the energy equation described by equation (A14) becomes the leading order (zero order) solution with respect to ν and hence, mathematically, ν should be absent in this expression. However, since this equation is obtained by integrating equation (A10) in the transverse direction, ν comes naturally in equation (A14) through the convective thermal boundary condition. Now, the potential distribution for the patterned electrothermal flow is given by

$$\bar{\psi}=\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}\frac{\cosh\left\{\left(\dfrac{\bar{\kappa}_0^2}{1+(\beta_1-\beta_2)\xi\bar{T}_0-\beta_1\beta_2\xi^2\bar{T}_0}\right)^{1/2}\bar{y}\right\}}{\cosh\left(\dfrac{\bar{\kappa}_0^2}{1+(\beta_1-\beta_2)\xi\bar{T}_0-\beta_1\beta_2\xi^2\bar{T}_0}\right)^{1/2}}\tag{A15}$$

Using typical values of the involving parameters, one can show that $(\beta_1-\beta_2)\xi\ll 1$ and $\beta_1\beta_2\xi^2\ll 1$. Hence, the potential distribution is simplified and takes the following form

$$\bar{\psi}=\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}\frac{\cosh\{\bar{\kappa}_0\bar{y}\}}{\cosh(\bar{\kappa}_0)}\tag{A16}$$

Using the expansion of equation (A13), the set of governing equations are rewritten below

$$\left.\begin{aligned}&\frac{\partial\bar{u}}{\partial\bar{x}}+\frac{\partial\bar{v}}{\partial\bar{y}}=0\\ &\frac{d\bar{p}}{d\bar{x}}=\frac{\partial}{\partial\bar{y}}\left\{\exp(-\xi\bar{T}_0)\bar{\tau}_{yx}\right\}-\bar{\kappa}_o^2\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}\frac{\cosh(\bar{\kappa}_0\bar{y})}{\cosh(\bar{\kappa}_0)}\frac{d\bar{\phi}}{d\bar{x}}\\ &\frac{1}{2}Pe\frac{d\bar{T}_0}{d\bar{x}}\left\{\int_{-1}^{1}\bar{u}\,d\bar{y}\right\}=\chi\frac{d^2\bar{T}_0}{d\bar{x}^2}-\frac{\nu\bar{T}_0}{\chi}+(1+\xi\beta_4\bar{T}_0)\left(\frac{d\bar{\phi}}{d\bar{x}}\right)^2\\ &\text{and}\qquad\frac{\partial}{\partial\bar{x}}\left\{(1+\xi\beta_4\bar{T}_0)\frac{d\bar{\phi}}{d\bar{x}}\right\}=0\end{aligned}\right\}\tag{A17}$$

The simplified stress components (after substituting $\bar{\tau}_{yy}=0$) are also expanded in a similar way

$$\frac{\partial\bar{u}}{\partial\bar{y}}=\left\{1+\frac{2\delta De_{\bar{\kappa}_o}^2}{\bar{\kappa}_0^2}\exp(-2\xi\bar{T}_0)(\bar{\tau}_{yx})\right\}\bar{\tau}_{yx}\tag{A18}$$

In order to solve the equations (A17)-(A18), we have used an asymptotic approach which is described in detail in **Section B** of the supplementary material.

Limiting Cases

On the basis of the present asymptotic analysis, we investigate some limiting cases.
Case 1: If we substitute $\xi=0$, the velocity profile reduces to the following form:

$$\begin{aligned}\bar{u}=&\frac{1}{2}\frac{d\bar{p}_{0,0}}{d\bar{x}}(\bar{y}^2-1)+\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}\left[1-\frac{\cosh(\bar{\kappa}_0\bar{y})}{\cosh(\bar{\kappa}_0)}\right]+\frac{1}{2}\frac{d\bar{p}_{0,1}}{d\bar{x}}(\bar{y}^2-1)\\ &+\frac{2\delta De^2}{\bar{\kappa}_0^2}\left[\begin{aligned}&\frac{a_1^3}{4}\left\{\cos(\omega\bar{x})-\frac{\sin(\omega)}{\omega}\right\}^3(\bar{y}^4-1)-\frac{\bar{\kappa}_0^2}{12}\frac{\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}^3}{\cosh^3(\bar{\kappa}_0)}f_3(\bar{y})\\ &-\frac{3a_1^2\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}}{\bar{\kappa}_0^2\cosh(\bar{\kappa}_0)}\left\{\cos(\omega\bar{x})-\frac{\sin(\omega)}{\omega}\right\}^2 f_1(\bar{y})\\ &+\frac{3a_1\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}^2}{8\cosh^2(\bar{\kappa}_0)}\left\{\cos(\omega\bar{x})-\frac{\sin(\omega)}{\omega}\right\}f_2(\bar{y})\end{aligned}\right]\end{aligned}\tag{A19}$$

which represents the flow field for a patterned electroosmotic flow of a viscoelastic fluid in absence of any thermal perturbation with the coefficients given in **Section C** of the supplementary material. Further simplification is possible by substituting $\alpha_2 = 0,\ \alpha_1 = 1$

$$\bar{u} = \left[1 - \frac{\cosh(\bar{\kappa}_0\,\bar{y})}{\cosh(\bar{\kappa}_0)}\right] - \frac{\delta\, De^2}{6\cosh^3(\bar{\kappa}_0)}\left[\cosh(3\bar{\kappa}_0\,\bar{y}) - 9\cosh(\bar{\kappa}_0\,\bar{y}) - \cosh(3\bar{\kappa}_0) + 9\cosh(\bar{\kappa}_0)\right] \quad \text{(A20)}$$

which is the velocity profile for purely electroosmotic flow of viscoelastic fluid through parallel plate microchannel. [5]

Case 2: If we substitute $De^* = 0$, the velocity profile reduces to the following form:

$$\begin{aligned}\bar{u} &= \frac{1}{2}\frac{d\bar{p}_{0,0}}{d\bar{x}}\left(\bar{y}^2 - 1\right) + \left\{\alpha_1 + \alpha_2\cos(\omega\bar{x})\right\}\left[1 - \frac{\cosh(\bar{\kappa}_0\,\bar{y})}{\cosh(\bar{\kappa}_0)}\right] \\ &+ \xi\left[\begin{array}{l} \frac{1}{2}\frac{d\bar{p}_{1,0}}{d\bar{x}}\left(\bar{y}^2 - 1\right) + \frac{1}{2}\bar{T}_{0,0,0}\frac{d\bar{p}_{0,0}}{d\bar{x}}\left(\bar{y}^2 - 1\right) + \left\{\alpha_1 + \alpha_2\cos(\omega\bar{x})\right\}\left\{\frac{\cosh(\bar{\kappa}_0\,\bar{y})}{\cosh(\bar{\kappa}_0)} - 1\right\}\frac{d\bar{\phi}_{1,0}}{d\bar{x}} \\ \qquad + \bar{T}_{0,0,0}\left\{\alpha_1 + \alpha_2\cos(\omega\bar{x})\right\}\left\{1 - \frac{\cosh(\bar{\kappa}_0\,\bar{y})}{\cosh(\bar{\kappa}_0)}\right\} \end{array}\right]\end{aligned} \quad \text{(A21)}$$

which is the velocity distribution for patterned electro-thermal flow of a Newtonian fluid. This is further reduced to, on substitution of $\alpha_2 = 0,\ \alpha_1 = 1$

$$\bar{u} = \left[1 - \frac{\cosh(\bar{\kappa}_0\,\bar{y})}{\cosh(\bar{\kappa}_0)}\right] + \xi\left[\left\{\frac{\cosh(\bar{\kappa}_0\,\bar{y})}{\cosh(\bar{\kappa}_0)} - 1\right\}\frac{d\bar{\phi}_{1,0}}{d\bar{x}} + \bar{T}_{0,0,0}\left\{1 - \frac{\cosh(\bar{\kappa}_0\,\bar{y})}{\cosh(\bar{\kappa}_0)}\right\}\right] \quad \text{(A22)}$$

The expressions for $\bar{T}_{0,0,0}$ and $d\bar{\phi}_{1,0}/d\bar{x}$ can be found in **Section F** of the Supplementary Material.

A1: Assumptions behind the Poisson-Boltzmann distribution

While obtaining the charge distribution, we have assumed that the Poisson-Boltzmann description remains valid. This assumption is based on the fact that ions are point charges and they are in local equilibrium. Under this condition, one can neglect the contribution of the advection term in the Poisson-Nernst-Plank equation when the value of ionic Peclet number Pe_i is very small as compared to unity $(Pe_i \ll 1)$. From definition, Pe_i is written as $Pe_i = u_{ref}\, h/D$ where u_{ref} is the characteristic velocity scale defined by $u_{ref} \sim \varepsilon_{ref}\zeta_{ref}^2/\mu_{ref} h$. In typical microfluidic applications, $\varepsilon_{ref} \sim 10^{-10}\ \mathrm{CV^{-1}\,m^{-1}}$, $\zeta_{ref} \sim 10^{-2}\ \mathrm{V}$, $\mu_{ref} \sim 10^{-3}\ \mathrm{Pa.s}$, and $D \sim 10^{-9}\ \mathrm{m^2 s^{-1}}$ [6] which yields $Pe_i \sim \mathrm{O}(10^{-6}) \ll 1$. In this context, it is also necessary to mention that the Poisson-Boltzmann description of the charge distribution breaks down in presence of finite sized ionic species in which one needs to take into account a more realistic model, commonly termed as modified Poisson-Boltzmann equation. [7–9] Accordingly, the present analysis is valid only when the effect of ionic mobility and finite size (also known as steric factor) are neglected.

B: Asymptotic Solution of Equations (A17)-(A18)

To obtain the asymptotic solution, we have used the well known regular perturbation technique where ξ is chosen as the gauge function to show the effect of thermal perturbation in the flow and temperature distribution. As already discussed, ξ depends strongly on the induced temperature difference (ΔT) and the temperature sensitivity parameter (α_6). To get a physical relevance, we have first extracted the data points of the viscosity reduction as a function of temperature, as reported by Huang and Yang. [10] Then, regression analysis of this data points is performed where these are fitted in the form of $\mu/\mu_{eff} = \alpha_\mu \cdot \exp\left\{-\alpha_6\left(T - T_{ref}\right)\right\}$ (this form of viscosity variation is used in the mathematical formulation) and the values of the fitting parameters are obtained as $\alpha_\mu = 0.9768$ and $\alpha_6 = 0.0175\,\mathrm{K}^{-1}$ respectively. As already discussed, the maximum ΔT is ~ 20 K up to which linear dependence with temperature is observed. Thus, choosing $\alpha_6 = 0.0175$ K^{-1} and $\Delta T \le 20$ K implies that the maximum possible value of the perturbation parameter can be chosen up to 0.35 while for small ΔT, this is obviously a very small quantity.

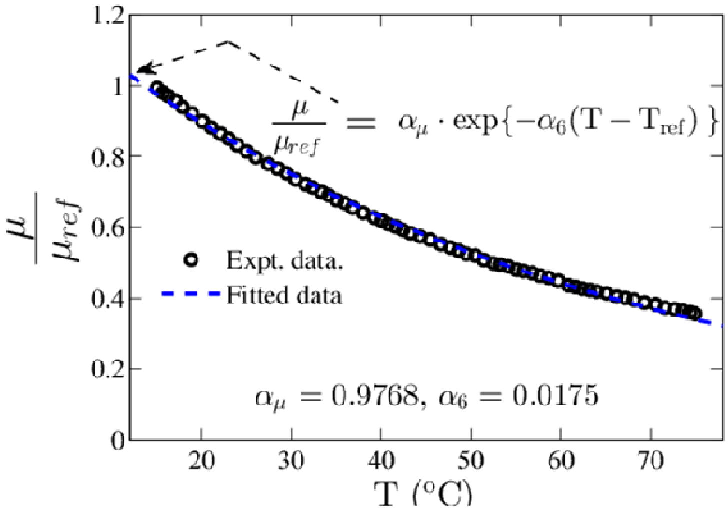

Figure 1. Normalised variation of viscosity (of a liquid having similar dependence of physical properties like water) with temperature.[10] Symbols represent the data points reported by Huang and Yang while solid line shows the fitted curve according to our functional relationship.

In the limit of $\xi \to 0$, any variable γ can be expanded in the following way

$$\gamma = \gamma_0 + \xi\,\gamma_1 + \xi^2\,\gamma_2 + \cdots\cdots \tag{B1}$$

For leading order, i.e. $\mathrm{O}\left(\xi^0\right)$, the set of equations are given below

$$\left.\begin{aligned}
&\frac{\partial \bar{u}_0}{\partial \bar{x}} + \frac{\partial \bar{v}_0}{\partial \bar{y}} = 0 \\
&\frac{d\bar{p}_0}{d\bar{x}} = \frac{\partial \bar{\tau}_{xy,0}}{\partial \bar{y}} - \bar{\kappa}_0^2 \left\{\alpha_1 + \alpha_2 \cos(\omega \bar{x})\right\} \frac{\cosh(\bar{\kappa}_0 \bar{y})}{\cosh(\bar{\kappa}_0)} \frac{d\bar{\phi}_0}{d\bar{x}} \\
&\frac{1}{2} Pe \frac{d\bar{T}_{0,0}}{d\bar{x}} \left\{\int_{-1}^{1} \bar{u}_0 \, d\bar{y}\right\} = \chi \frac{d^2 \bar{T}_{0,0}}{d\bar{x}^2} - \frac{\nu \bar{T}_{0,0}}{\chi} + \left(\frac{d\bar{\phi}_0}{d\bar{x}}\right)^2 \\
&\frac{d^2 \phi_0}{d\bar{x}^2} = 0
\end{aligned}\right\} \tag{B2}$$

and $$\frac{\partial \bar{u}_0}{\partial \bar{y}} = \left\{1 + \frac{2\delta De_{\bar{\kappa}_0}^2}{\bar{\kappa}_0^2} \bar{\tau}_{yx,0}^2\right\} \bar{\tau}_{yx,0} \tag{B3}$$

For first order, i.e. $O(\xi^1)$,

$$\left.\begin{aligned}
&\frac{\partial \bar{u}_1}{\partial \bar{x}} + \frac{\partial \bar{v}_1}{\partial \bar{y}} = 0 \\
&\frac{d\bar{p}_1}{d\bar{x}} = \frac{\partial \bar{\tau}_{xy,1}}{\partial \bar{y}} - \bar{T}_{0,0} \frac{\partial \bar{\tau}_{xy,0}}{\partial \bar{y}} - \bar{\kappa}_0^2 \left\{\alpha_1 + \alpha_2 \cos(\omega \bar{x})\right\} \frac{\cosh(\bar{\kappa}_0 \bar{y})}{\cosh(\bar{\kappa}_0)} \frac{d\bar{\phi}_1}{d\bar{x}} \\
&\beta_4 \frac{d}{d\bar{x}} \left(\bar{T}_{0,0} \frac{d\bar{\phi}_0}{d\bar{x}}\right) + \frac{d^2 \bar{\phi}_1}{d\bar{x}^2} = 0
\end{aligned}\right\} \tag{B4}$$

and $$\frac{\partial \bar{u}_1}{\partial \bar{y}} = \bar{\tau}_{yx,1} + \frac{6\delta De_{\bar{\kappa}_0}^2}{\bar{\kappa}_0^2} \bar{\tau}_{yx,0}^2 \, \bar{\tau}_{yx,1} \tag{B5}$$

The boundary conditions described by equation (A6) are rewritten in the following manner:

$$\left.\begin{aligned}
&\bar{u}_0(\bar{y} = \pm 1) = \bar{u}_1(\bar{y} = \pm 1) = 0, \\
&\bar{v}_0(\bar{y} = \pm 1) = \bar{v}_1(\bar{y} = \pm 1) = 0, \\
&\bar{p}_0(\bar{x} = 0,1) = \bar{p}_1(\bar{x} = 0,1) = 0, \\
&\bar{T}_{0,0}(x = 0,1) = \bar{T}_{0,1}(\bar{x} = 0,1) = 0, \\
&\bar{\phi}_0(x = 0) = 1, \ \bar{\phi}_1(\bar{x} = 0) = 0, \\
&\bar{\phi}_0(x = 1) = 0, \ \bar{\phi}_1(\bar{x} = 1) = 0.
\end{aligned}\right\} \tag{B6}$$

Now, the stress component used in equation (B3) is substituted by the following expression

$$\bar{\tau}_{xy,0} = \frac{d\bar{p}_0}{d\bar{x}} \bar{y} - \bar{\kappa}_0 \left\{\alpha_1 + \alpha_2 \cos(\omega \bar{x})\right\} \frac{\sinh(\bar{\kappa}_0 \bar{y})}{\cosh(\bar{\kappa}_0)} \tag{B7}$$

where symmetry condition is taken into account at the channel centreline, i.e. $\bar{\tau}_{xy,0}(\bar{y} = 0) = 0$. Hence, the simplified momentum equation in the leading order takes the form

$$\frac{\partial \bar{u}_0}{\partial \bar{y}}=\frac{d\bar{p}_0}{d\bar{x}}\bar{y}-\bar{\kappa}_0^2\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}\frac{\sinh(\bar{\kappa}_0\bar{y})}{\cosh(\bar{\kappa}_0)}$$
$$+\frac{2\delta}{\bar{\kappa}_0^2}De_{\bar{\kappa}_0}^2\left\{\frac{d\bar{p}_0}{d\bar{x}}\bar{y}-\bar{\kappa}_0^2\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}\frac{\sinh(\bar{\kappa}_0\bar{y})}{\cosh(\bar{\kappa}_0)}\right\}^3 \tag{B8}$$

To solve this non-linear equation, one needs to use the same asymptotic approach and the variables are expanded in a similar fashion using De^* as the perturbation parameter defined as $De^*=De^2$. Hence, all $\mathrm{O}(1)$ terms represent the Newtonian contribution while $\mathrm{O}(De^*)$ and higher order terms are showing the viscoelastic counterpart. Now, the variables are expanded in the following way

$$\left.\begin{aligned}
\bar{u}_0&=\bar{u}_{0,0}+De^*\,\bar{u}_{0,1}+De^{*2}\,\bar{u}_{0,2}+\cdots\cdots\\
\bar{v}_0&=\bar{v}_{0,0}+De^*\,\bar{v}_{0,1}+De^{*2}\,\bar{v}_{0,2}+\cdots\cdots\\
\bar{p}_0&=\bar{p}_{0,0}+De^*\bar{p}_{0,1}+De^{*2}\,\bar{p}_{0,2}+\cdots\cdots\\
\bar{T}_0&=\bar{T}_{0,0}+De^*\bar{T}_{0,1}+De^{*2}\,\bar{T}_{0,2}+\ldots.\\
\bar{\phi}_0&=\bar{\phi}_{0,0}+De^*\,\bar{\phi}_{0,1}+De^{*2}\,\bar{\phi}_{0,2}+\ldots.
\end{aligned}\right\}$$

Here, all terms with subscript 0,0 represents the leading order solution, i.e. the Newtonian contribution part while subscripts like 0,1 and 0,2 correspond to their viscoelastic counterpart Now we expand the variables of equation (B8) and the equations are given below

For $\mathrm{O}(1)$:
$$\frac{\partial \bar{u}_{0,0}}{\partial \bar{y}}=\frac{d\bar{p}_{0,0}}{d\bar{x}}\bar{y}-\bar{\kappa}_0^2\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}\frac{\sinh(\bar{\kappa}_0\bar{y})}{\cosh(\bar{\kappa}_0)} \tag{B9}$$

For $\mathrm{O}(De^*)$:
$$\frac{\partial \bar{u}_{0,1}}{\partial \bar{y}}=\frac{d\bar{p}_{0,1}}{d\bar{x}}\bar{y}+\frac{2\delta}{\bar{\kappa}_0^2}\begin{bmatrix}\bar{y}^3\left(\dfrac{d\bar{p}_{0,0}}{d\bar{x}}\right)^3-\bar{\kappa}_0^3\dfrac{\sinh^3(\bar{\kappa}_0\bar{y})}{\cosh^3(\bar{\kappa}_0)}\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}^3\\ +3\bar{\kappa}_0^2\,\bar{y}\dfrac{\sinh^2(\bar{\kappa}_0\bar{y})}{\cosh^2(\bar{\kappa}_0)}\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}^2\left(\dfrac{d\bar{p}_{0,0}}{d\bar{x}}\right)\\ -3\bar{\kappa}_0\,\bar{y}^2\dfrac{\sinh(\bar{\kappa}_0\bar{y})}{\cosh(\bar{\kappa}_0)}\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}\left(\dfrac{d\bar{p}_{0,0}}{d\bar{x}}\right)^2\end{bmatrix} \tag{B10}$$

Now the solution of equation (B9) subjected to the no-slip boundary condition is given by

$$\bar{u}_{0,0}=\frac{1}{2}\frac{d\bar{p}_{0,0}}{d\bar{x}}\left(\bar{y}^2-1\right)+\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}\left[1-\frac{\cosh(\bar{\kappa}_0\bar{y})}{\cosh(\bar{\kappa}_0)}\right] \tag{B11}$$

where the pressure gradient $d\bar{p}_{0,0}/d\bar{x}$ is yet to be determined. This can be done by invoking the continuity equation to determine the *v*-component of the flow field which is then subjected to the impermeability condition at the surfaces $\bar{v}_0(\bar{y}=\pm1)=0$ and yields

$$\bar{p}_{0,0}=\frac{3\alpha_2}{\omega}\left[1-\frac{\tanh(\bar{\kappa}_0)}{\bar{\kappa}_0}\right]\{\sin(\omega\bar{x})-\bar{x}\sin(\omega)\} \tag{B12}$$

and $$\frac{d\bar{p}_{0,0}}{d\bar{x}}=3\alpha_2\left[1-\frac{\tanh(\bar{\kappa}_0)}{\bar{\kappa}_0}\right]\left\{\cos(\omega\bar{x})-\frac{\sin(\omega)}{\omega}\right\} \tag{B13}$$

Similarly proceeding, the solution of equation (B10) is given by

$$\bar{u}_{0,1}=\frac{1}{2}\frac{d\bar{p}_{0,1}}{d\bar{x}}\left(\bar{y}^2-1\right)+\frac{2\delta}{\bar{\kappa}_0^2}\left[\begin{array}{l}\frac{a_1^3}{4}\left\{\cos(\omega\bar{x})-\frac{\sin(\omega)}{\omega}\right\}^3\left(\bar{y}^4-1\right)-\frac{\bar{\kappa}_0^2}{12}\frac{\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}^3}{\cosh^3(\bar{\kappa}_0)}f_3(\bar{y})\\ -\frac{3a_1^2\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}}{\bar{\kappa}_0^2\cosh(\bar{\kappa}_0)}\left\{\cos(\omega\bar{x})-\frac{\sin(\omega)}{\omega}\right\}^2 f_1(\bar{y})\\ +\frac{3a_1\{\alpha_1+\alpha_2\cos(\omega\bar{x})\}^2}{8\cosh^2(\bar{\kappa}_0)}\left\{\cos(\omega\bar{x})-\frac{\sin(\omega)}{\omega}\right\}f_2(\bar{y})\end{array}\right] \tag{B14}$$

along with the pressure distribution

$$\bar{p}_{0,1}=\frac{3\delta}{\bar{\kappa}_0^2}\left[\begin{array}{l}-\frac{a_1^3}{15\omega^3}f_1(\bar{x})+\frac{a_2 f_1(\bar{\kappa}_0)\alpha_2}{2\bar{\kappa}_0\ \omega^3}f_2(\bar{x})+\frac{a_3 f_1(\bar{\kappa}_0)}{18\ \bar{\kappa}_0\ \omega^2}f_3(\bar{x})\\ -\frac{a_4 f_2(\bar{\kappa}_0)}{27\ \bar{\kappa}_0\ \alpha_2\,\omega}f_4(\bar{x})-\frac{a_5 f_2(\bar{\kappa}_0)\alpha_2}{3\ \bar{\kappa}_0\,\omega^2}f_5(\bar{x})-\frac{8a_6 f_3(\bar{\kappa}_0)\alpha_2}{\bar{\kappa}_0\,\omega}f_6(\bar{x})\end{array}\right]+c_1\bar{x} \tag{B15}$$

The coefficients of equations (B12)-(B15) are given in **Section C**. Once the velocity distribution is known, one can evaluate the corresponding temperature and potential distribution

$$\bar{T}_{0,0}=\frac{\frac{\chi}{\nu}\{\exp(d_1)-1\}}{\exp(d_2)-\exp(d_1)}\{\exp(d_2\bar{x})-\exp(d_1\bar{x})\}+\frac{\chi}{\nu}\{1-\exp(d_1\bar{x})\} \tag{B16}$$

and $$\bar{\phi}_0=1-\bar{x} \tag{B17}$$

The coefficients of equation (B16) can be found in **Section D**.

Knowing the leading order temperature and potential distribution, one can calculate the higher order potential distribution, as evident from equation (B4)

$$\bar{\phi}_1=\beta_4\left[\frac{\frac{\chi}{\nu}\{\exp(d_1)-1\}}{\exp(d_2)-\exp(d_1)}\right]\left\{\frac{\exp(d_2\bar{x})-\bar{x}\exp(d_2)+\bar{x}-1}{d_2}+\frac{1-\bar{x}}{d_1}+\frac{\bar{x}\exp(d_1)-\exp(d_1\bar{x})}{d_1}\right\}$$
$$+\frac{\beta_4\chi}{\nu}\left\{\frac{\bar{x}\exp(d_1)-\exp(d_1\bar{x})-\bar{x}+1}{d_1}\right\} \tag{B18}$$

and

$$\frac{d\bar{\phi}_1}{d\bar{x}}{}_1=\beta_4\left[\frac{\frac{\chi}{\nu}\{\exp(d_1)-1\}}{\exp(d_2)-\exp(d_1)}\right]\left\{\frac{1-\exp(d_2)}{d_2}+\frac{\exp(d_1)-1}{d_1}+\exp(d_2\bar{x})-\exp(d_1\bar{x})\right\}$$
$$+\frac{\beta_4\chi}{\nu}\left\{\frac{\exp(d_1)-1}{d_1}-\exp(d_1\bar{x})\right\} \tag{B19}$$

Once $\bar{\phi}_0$ and $\bar{\phi}_1$ are known, the electrothermal body force $\left(\bar{F}_x\right)$ in the momentum equation can be evaluated as

$$\bar{F}_x = -\frac{1}{2\bar{\kappa}_0}\int_{-1}^{1}\bar{\kappa}_0^2\left\{\alpha_1+\alpha_2\cos\left(\omega\bar{x}\right)\right\}\frac{\cosh\left(\bar{\kappa}_0\,\bar{y}\right)}{\cosh\left(\bar{\kappa}_0\right)}\frac{d\bar{\phi}}{d\bar{x}}\,d\,\bar{y} \tag{B20}$$

The solution for the set of equations of $\mathrm{O}\left(\xi^1\right)$ described by equations (B4)-(B5) are presented in **Section E**. Once the solution is obtained, volumetric flow rate $\bar{Q}$ through the microchannel can be calculated as

$$\begin{aligned}\bar{Q} &= \int_{-1}^{1}\bar{u}\;d\bar{y} = \int_{-1}^{1}\left(\bar{u}_0+\xi\,\bar{u}_1\right)\,d\bar{y}\\ &= \int_{-1}^{1}\left(\bar{u}_{0,0}+De^{*}\bar{u}_{0,1}+\xi\,\bar{u}_1+\mathrm{O}\left(\xi^2\right)+\mathrm{O}\left(De^{*2}\right)+\mathrm{O}\left(\xi De^{*}\right)+....\right)\,d\bar{y}\end{aligned} \tag{B21}$$

where the solution is presented corrected up to the first order $\mathrm{O}\left(\xi^1\right)$ because of the inherent non-linearity of the governing equations.

C: The coefficients of Equations (B12)-(B15)

$$a_1 = 3\alpha_2\left[1-\frac{\tanh\left(\bar{\kappa}_0\right)}{\bar{\kappa}_0}\right], a_2 = a_3 = \frac{3a_1^2}{\bar{\kappa}_0^2\cosh\left(\bar{\kappa}_0\right)}, a_4 = a_5 = \frac{3a_1}{8\cosh^2\left(\bar{\kappa}_0\right)}, a_6 = \frac{\bar{\kappa}_0^2}{12\cosh^3\left(\bar{\kappa}_0\right)} \tag{C1}$$

$$a_1 = 3\alpha_2\left[1-\frac{\tanh\left(\bar{\kappa}_0\right)}{\bar{\kappa}_0}\right], a_2 = a_3 = \frac{3a_1^2}{\bar{\kappa}_0^2\cosh\left(\bar{\kappa}_0\right)}, a_4 = a_5 = \frac{3a_1}{8\cosh^2\left(\bar{\kappa}_0\right)}, a_6 = \frac{\bar{\kappa}_0^2}{12\cosh^3\left(\bar{\kappa}_0\right)} \tag{C2}$$

$$\begin{aligned}f_1\left(\bar{y}\right) &= \left[\left(\bar{\kappa}_0^2\bar{y}^2+2\right)\cosh\left(\bar{\kappa}_0\,\bar{y}\right)-2\,\bar{\kappa}_0\,\bar{y}\sinh\left(\bar{\kappa}_0\,\bar{y}\right)-\left(\bar{\kappa}_0^2+2\right)\cosh\left(\bar{\kappa}_0\right)+2\,\bar{\kappa}_0\sinh\left(\bar{\kappa}_0\right)\right]\\ f_2\left(\bar{y}\right) &= \left[2\,\bar{\kappa}_0\,\bar{y}\left\{\sinh\left(2\,\bar{\kappa}_0\,\bar{y}\right)-\bar{\kappa}_0\,\bar{y}\right\}-\cosh\left(2\,\bar{\kappa}_0\,\bar{y}\right)-2\,\bar{\kappa}_0\left\{\sinh\left(2\,\bar{\kappa}_0\right)-\bar{\kappa}_0\right\}+\cosh\left(2\,\bar{\kappa}_0\right)\right]\\ f_3\left(\bar{y}\right) &= \left[\cosh\left(3\,\bar{\kappa}_0\,\bar{y}\right)-9\cosh\left(\bar{\kappa}_0\,\bar{y}\right)-\cosh\left(3\,\bar{\kappa}_0\right)+9\cosh\left(\bar{\kappa}_0\right)\right]\end{aligned} \tag{C3}$$

$$\begin{aligned}f_1\left(\bar{\kappa}_0\right) &= \left[6\,\bar{\kappa}_0^2\sinh\left(\bar{\kappa}_0\right)-2\,\bar{\kappa}_0^3\cosh\left(\bar{\kappa}_0\right)-12\,\bar{\kappa}_0\cosh\left(\bar{\kappa}_0\right)+12\sinh\left(\bar{\kappa}_0\right)\right]\\ f_2\left(\bar{\kappa}_0\right) &= \left[-4\,\bar{\kappa}_0^3+6\,\bar{\kappa}_0^2\sinh\left(2\,\bar{\kappa}_0\right)-6\,\bar{\kappa}_0\cosh\left(2\,\bar{\kappa}_0\right)+3\sinh\left(2\,\bar{\kappa}_0\right)\right]\\ f_3\left(\bar{\kappa}_0\right) &= \left[-3\,\bar{\kappa}_0\cosh^3\left(\bar{\kappa}_0\right)+\sinh\left(\bar{\kappa}_0\right)\cosh^2\left(\bar{\kappa}_0\right)+9\,\bar{\kappa}_0\cosh\left(\bar{\kappa}_0\right)-7\sinh\left(\bar{\kappa}_0\right)\right]\end{aligned} \tag{C4}$$

$$\begin{aligned}f_1\left(\bar{x}\right) &= \left[\left\{2\omega^2\cos^2\left(\omega\bar{x}\right)+4\omega^2-9\omega\cos\left(\omega\bar{x}\right)\sin\left(\omega\right)+18-18\cos^2\left(\omega\right)\right\}\sin\left(\omega\bar{x}\right)+9\omega^2\bar{x}\sin\left(\omega\right)\right]\\ f_2\left(\bar{x}\right) &= \left[\left\{-\frac{2}{9}\omega^2\cos^2\left(\omega\bar{x}\right)+\omega\sin\left(\omega\right)\cos\left(\omega\bar{x}\right)-\frac{4}{9}\omega^2+2\cos^2\left(\omega\right)-2\right\}\sin\left(\omega\bar{x}\right)\right.\\ &\quad\left.+\bar{x}\sin\left(\omega\right)\left\{\omega^2-\frac{2}{3}\cos^2\left(\omega\right)+\frac{2}{3}\right\}\right]\end{aligned} \tag{C5}$$

$$\begin{aligned}f_3\left(\bar{x}\right) &= \left[\left\{\begin{matrix}4\alpha_2\,\omega\cos^2\left(\omega\right)+\left(9A\omega-9\alpha_2\sin\omega\right)\cos\left(\omega\bar{x}\right)\\ -36\alpha_1\sin\left(\omega\right)+8\alpha_2\,\omega\end{matrix}\right\}\sin\left(\omega\bar{x}\right)+9\,\omega\bar{x}\left\{\alpha_1\,\omega-\alpha_2\sin\left(\omega\right)\right\}\right]\\ f_4\left(\bar{x}\right) &= \left[\left\{2\alpha_2^3\cos^2\left(\omega\bar{x}\right)+9\alpha_1\,\alpha_2^2\cos\left(\omega\bar{x}\right)+18\alpha_1^2\,\alpha_2+4\alpha_2^3\right\}\sin\left(\omega\bar{x}\right)+6\alpha_1\,\omega\left(\alpha_1^2+\frac{3}{2}\alpha_2^2\right)\bar{x}\right]\end{aligned} \tag{C6}$$

$$f_5(\bar{x}) = \left[\left\{ \begin{array}{l} \frac{4}{9}\alpha_2\,\omega\cos^2(\omega\bar{x}) + (\alpha_1\,\omega - \alpha_2\sin\omega)\cos(\omega\bar{x}) \\ -4\alpha_1\sin(\omega) + \frac{8}{9}\alpha_2\,\omega \end{array} \right\} \sin(\omega\bar{x}) + \omega\bar{x}\{\alpha_1\,\omega - \alpha_2\sin(\omega)\} \right] \tag{C7}$$

$$f_6(\bar{x}) = \left[\left\{ \frac{1}{9}\alpha_2^{\,2}\cos^2(\omega\bar{x}) + \frac{1}{2}\alpha_1\,\alpha_2\cos(\omega\bar{x}) + \alpha_1^{\,2} + \frac{2}{9}\alpha_2^{\,2} \right\} \sin(\omega\bar{x}) - \frac{1}{2}\alpha_1\,\alpha_2\,\omega\bar{x} \right]$$

$$c_1 = \left(-\frac{3\delta}{\bar{\kappa}_0^2} \right) \left[\begin{array}{l} -\frac{a_1^3}{15\,\omega^3} f_1(\omega) + \frac{a_2\, f_1(\bar{\kappa}_0)\,\alpha_2}{2\,\bar{\kappa}_0\,\omega^3} f_2(\omega) + \frac{a_3\, f_1(\bar{\kappa}_0)}{18\,\bar{\kappa}_0\,\omega^2} f_3(\omega) \\ -\frac{a_4\, f_2(\bar{\kappa}_0)}{27\,\bar{\kappa}_0\,\alpha_2\,\omega} f_4(\omega) - \frac{a_5\, f_2(\bar{\kappa}_0)\,\alpha_2}{3\,\bar{\kappa}_0\,\omega^2} f_5(\omega) - \frac{8\,a_6\, f_3(\bar{\kappa}_0)\,\alpha_2}{\bar{\kappa}_0\,\omega} f_6(\omega) \end{array} \right] \tag{C8}$$

$$\begin{aligned} f_1(\omega) &= \left[\left\{ (2\omega^2 - 18)\cos^2(\omega) + 4\,\omega^2 - 9\,\omega\cos(\omega)\sin(\omega) + 18 \right\} \sin(\omega) + 9\,\omega^2\sin(\omega) \right] \\ f_2(\omega) &= \left[\left\{ 2\omega^2\cos^2(\omega) - 9\,\omega\sin(\omega)\cos(\omega) - 5\,\omega^2 - 12\cos^2(\omega) + 12 \right\} \sin(\omega) \right] \end{aligned} \tag{C9}$$

$$\begin{aligned} f_3(\omega) &= \left[\{-9\alpha_2\cos(\omega) - 36\alpha_1\}\sin^2(\omega) + 9\,\omega\sin(\omega) \left\{ \frac{4\alpha_2}{9}\cos^2(\omega) + \alpha_1\cos(\omega) - \frac{\alpha_2}{9} + 9\alpha_1\,\omega^2 \right\} \right] \\ f_4(\omega) &= \left[\left\{ 2\alpha_2^{\,3}\cos^2(\omega\bar{x}) + 9\,\alpha_1\,\alpha_2^{\,2}\cos(\omega\bar{x}) + 18\alpha_1^{\,2}\,\alpha_2 + 4\alpha_2^{\,3} \right\} \sin(\omega\bar{x}) + 6\alpha_1\,\omega \left(\alpha_1^{\,2} + \frac{3}{2}\alpha_2^{\,2} \right) \bar{x} \right] \end{aligned} \tag{C10}$$

$$\begin{aligned} f_5(\omega) &= \left[\left\{ \frac{4}{9}\alpha_2\cos^2(\omega) + \alpha_1\cos(\omega) - \frac{\alpha_2}{9} \right\} \omega\sin(\omega) + \alpha_1\,\omega^2 + \sin^2(\omega)\{-\alpha_2\cos(\omega) - 4\alpha_1\} \right] \\ f_6(\omega) &= \left[\left\{ \frac{1}{9}\alpha_2^{\,2}\cos^2(\omega) + \frac{1}{2}\alpha_1\,\alpha_2\cos(\omega) + \alpha_1^{\,2} + \frac{2}{9}\alpha_2^{\,2} \right\} \sin(\omega) - \frac{1}{2}\alpha_1\,\alpha_2\,\omega \right] \end{aligned} \tag{C11}$$

D: The coefficients of Equation (B16)

$$d_1 = \frac{\left[b_1\,Pe_T + \sqrt{b_1^2 Pe_T^2 + 4\nu} \right]}{2\chi}, \quad d_2 = \frac{\left[b_1\,Pe_T - \sqrt{b_1^2 Pe_T^2 + 4\nu} \right]}{2\chi} \quad \text{and} \quad b_1 = \frac{1}{2}\int_{-1}^{1} \left(\bar{u}_{0,0} + De^*\bar{u}_{0,1} \right) d\bar{y}$$

where $2b_1$ represents the leading order volumetric flow rate through the microchannel i.e., in absence of any thermal perturbation.

E: The solution of Equations (B4)-(B5)

First, we have obtained the stress component of the first order which in turn is used to determine the flow field

$$\begin{aligned} \bar{\tau}_{xy,1} = \frac{d\bar{p}_1}{d\bar{x}}\bar{y} + \bar{T}_{0,0}\frac{d\bar{p}_0}{d\bar{x}}\bar{y} + \bar{\kappa}_0\{\alpha_1 + \alpha_2\cos(\omega\bar{x})\}\frac{\sinh(\bar{\kappa}_0\bar{y})}{\cosh(\bar{\kappa}_0)}\frac{d\bar{\phi}_1}{d\bar{x}} \\ -\bar{T}_{0,0}\,\bar{\kappa}_0\{\alpha_1 + \alpha_2\cos(\omega\bar{x})\}\frac{\sinh(\bar{\kappa}_0\bar{y})}{\cosh(\bar{\kappa}_0)} \end{aligned} \tag{E1}$$

$$\frac{\partial \bar{u}_1}{\partial \bar{y}} = \frac{d\bar{p}_1}{d\bar{x}}\bar{y} + \bar{T}_{0,0}\frac{d\bar{p}_0}{d\bar{x}}\bar{y} + \bar{\kappa}_0\left\{\alpha_1 + \alpha_2\cos\left(\omega\bar{x}\right)\right\}\frac{\sinh\left(\bar{\kappa}_0\bar{y}\right)}{\cosh\left(\bar{\kappa}_0\right)}\frac{d\bar{\phi}_1}{d\bar{x}}$$

$$-\bar{T}_{0,0}\,\bar{\kappa}_0\left\{\alpha_1 + \alpha_2\cos\left(\omega\bar{x}\right)\right\}\frac{\sinh\left(\bar{\kappa}_0\bar{y}\right)}{\cosh\left(\bar{\kappa}_0\right)}$$

$$+\frac{6\delta De_{\bar{\kappa}_0}^2}{\bar{\kappa}_0^2}\left[\frac{d\bar{p}_0}{d\bar{x}}\bar{y} - \bar{\kappa}_0\left\{\alpha_1 + \alpha_2\cos\left(\omega\bar{x}\right)\right\}\frac{\sinh\left(\bar{\kappa}_0\bar{y}\right)}{\cosh\left(\bar{\kappa}_0\right)}\right]^2\left[\begin{array}{l}\frac{d\bar{p}_1}{d\bar{x}}\bar{y} + \bar{T}_{0,0}\frac{d\bar{p}_0}{d\bar{x}}\bar{y} \\ +\bar{\kappa}_0\left\{\alpha_1 + \alpha_2\cos\left(\omega\bar{x}\right)\right\}\frac{\sinh\left(\bar{\kappa}_0\bar{y}\right)}{\cosh\left(\bar{\kappa}_0\right)}\frac{d\bar{\phi}_1}{d\bar{x}} \\ -\bar{T}_{0,0}\,\bar{\kappa}_0\left\{\alpha_1 + \alpha_2\cos\left(\omega\bar{x}\right)\right\}\frac{\sinh\left(\bar{\kappa}_0\bar{y}\right)}{\cosh\left(\bar{\kappa}_0\right)}\end{array}\right] \quad \text{(E2)}$$

To solve this equation (E2), we have again used the same approach and the variables are expanded as follows

$$\left.\begin{array}{l}\bar{u}_1 = \bar{u}_{1,0} + De^*\bar{u}_{1,1} + De^{*2}\,\bar{u}_{1,2} + \cdots\cdots \\ \bar{v}_1 = \bar{v}_{1,0} + De^*\bar{v}_{1,1} + De^{*2}\,\bar{v}_{1,2} + \cdots\cdots \\ \bar{p}_1 = \bar{p}_{1,0} + De^*\bar{p}_{1,1} + De^{*2}\,\bar{p}_{1,2} + \cdots\cdots \\ \bar{T}_{0,0} = \bar{T}_{0,0,0} + De^*\bar{T}_{0,0,1} + De^{*2}\,\bar{T}_{0,0,2} + \ldots. \\ \bar{\phi}_1 = \bar{\phi}_{1,0} + De^*\,\bar{\phi}_{1,1} + De^{*2}\,\bar{\phi}_{1,2} + \ldots.\end{array}\right\}$$

Thus, equation (E2) is splitted into two following set of equations

$$\begin{array}{l}\frac{\partial \bar{u}_{1,0}}{\partial \bar{y}} = \frac{d\bar{p}_{1,0}}{d\bar{x}}\bar{y} + \bar{T}_{0,0,0}\frac{d\bar{p}_{0,0}}{d\bar{x}}\bar{y} + \bar{\kappa}_0\left\{\alpha_1 + \alpha_2\cos\left(\omega\bar{x}\right)\right\}\frac{\sinh\left(\bar{\kappa}_0\,\bar{y}\right)}{\cosh\left(\bar{\kappa}_0\right)}\frac{d\bar{\phi}_{1,0}}{d\bar{x}} \\ \qquad -\bar{T}_{0,0,0}\,\bar{\kappa}_0\left\{\alpha_1 + \alpha_2\cos\left(\omega\bar{x}\right)\right\}\frac{\sinh\left(\bar{\kappa}_0\,\bar{y}\right)}{\cosh\left(\bar{\kappa}_0\right)}\end{array} \quad \text{(E3)}$$

$$\frac{\partial \bar{u}_{1,1}}{\partial \bar{y}} = \frac{d\bar{p}_{1,1}}{d\bar{x}}\bar{y} + \bar{T}_{0,0,0}\,\bar{y}\frac{d\bar{p}_{0,1}}{d\bar{x}} + \bar{T}_{0,0,1}\,\bar{y}\frac{d\bar{p}_{0,0}}{d\bar{x}}$$

$$+\bar{\kappa}_0\left\{\alpha_1 + \alpha_2\cos\left(\omega\bar{x}\right)\right\}\frac{\sinh\left(\bar{\kappa}_0\,\bar{y}\right)}{\cosh\left(\bar{\kappa}_0\right)}\frac{d\bar{\phi}_{1,1}}{d\bar{x}} - \bar{T}_{0,0,1}\bar{\kappa}_0\left\{\alpha_1 + \alpha_2\cos\left(\omega\bar{x}\right)\right\}\frac{\sinh\left(\bar{\kappa}_0\,\bar{y}\right)}{\cosh\left(\bar{\kappa}_0\right)}$$

$$+\frac{6\delta}{\bar{\kappa}_0^2}\left[\begin{array}{l}\frac{d\bar{p}_{0,0}}{d\bar{x}}\bar{y} - \\ \bar{\kappa}_0\left\{\alpha_1 + \alpha_2\cos\left(\omega\bar{x}\right)\right\}\frac{\sinh\left(\bar{\kappa}_0\,\bar{y}\right)}{\cosh\left(\bar{\kappa}_0\right)}\end{array}\right]^2\left[\begin{array}{c}\frac{d\bar{p}_{1,0}}{d\bar{x}}\bar{y} + \bar{T}_{0,0,0}\frac{d\bar{p}_{0,0}}{d\bar{x}}\bar{y} + \\ \bar{\kappa}_0\left\{\alpha_1 + \alpha_2\cos\left(\omega\bar{x}\right)\right\}\frac{\sinh\left(\bar{\kappa}_0\,\bar{y}\right)}{\cosh\left(\bar{\kappa}_0\right)}\frac{d\bar{\phi}_{1,0}}{d\bar{x}} \\ -\bar{T}_{0,0,0}\,\bar{\kappa}_0\left\{\alpha_1 + \alpha_2\cos\left(\omega\bar{x}\right)\right\}\frac{\sinh\left(\bar{\kappa}_0\,\bar{y}\right)}{\cosh\left(\bar{\kappa}_0\right)}\end{array}\right] \quad \text{(E4)}$$

The solution of equation (E3) is given by

$$\bar{u}_{1,0} = \frac{1}{2}\frac{d\bar{p}_{1,0}}{d\bar{x}}\left(\bar{y}^2-1\right) + \frac{1}{2}\bar{T}_{0,0,0}\frac{d\bar{p}_{0,0}}{d\bar{x}}\left(\bar{y}^2-1\right) + \left\{\alpha_1+\alpha_2\cos\left(\omega\bar{x}\right)\right\}\left\{\frac{\cosh\left(\bar{\kappa}_0\,\bar{y}\right)}{\cosh\left(\bar{\kappa}_0\right)}-1\right\}\frac{d\bar{\phi}_{1,0}}{d\bar{x}}$$
$$+\bar{T}_{0,0,0}\left\{\alpha_1+\alpha_2\cos\left(\omega\bar{x}\right)\right\}\left\{1-\frac{\cosh\left(\bar{\kappa}_0\,\bar{y}\right)}{\cosh\left(\bar{\kappa}_0\right)}\right\} \tag{E5}$$

where $d\bar{p}_{1,0}/d\bar{x}$ is obtained in a similar fashion as mentioned earlier. Finally the expression for the velocity profile can be given by

$$\begin{aligned}\bar{u} &= \bar{u}_{0,0} + De^{*}\bar{u}_{0,1} + De^{*2}\bar{u}_{0,2} + \xi\left(\bar{u}_{1,0} + De^{*}\bar{u}_{1,1}\right) + \ldots \\ &= \bar{u}_{0,0} + De^{*}\bar{u}_{0,1} + \xi\,\bar{u}_1 + \mathrm{O}\left(\xi^2\right) + \mathrm{O}\left(De^{*2}\right) + \mathrm{O}\left(\xi De^{*}\right) + \ldots\end{aligned} \tag{E6}$$

Here, the results are reported correct up to first order where the contributions from the higher order terms are omitted for simplification. The expressions of $\bar{T}_{0,0,0}$ and $d\bar{\phi}_{1,0}/d\bar{x}$ presented in **Section F** for the completeness of the problem.

F: The expressions of $\bar{T}_{0,0,0}$ and $d\bar{\phi}_{1,0}/d\bar{x}$

$$\bar{T}_{0,0,0} = \frac{\chi}{\nu}\cdot\frac{\left(\exp\left(d_{1,0}\right)-1\right)}{\left(\exp\left(d_{2,0}\right)-\exp\left(d_{1,0}\right)\right)}\cdot\left(\exp\left(d_{2,0}\,x\right)-\exp\left(d_{1,0}\,x\right)\right) + \frac{\chi}{\nu}\cdot\left(1-\exp\left(d_{1,0}\,x\right)\right) \tag{F1}$$

$$\frac{d\bar{\phi}_{1,0}}{d\bar{x}} = \beta_4\left[\frac{\frac{\chi}{\nu}\left\{\exp\left(d_{1,0}\right)-1\right\}}{\exp\left(d_{2,0}\right)-\exp\left(d_{1,0}\right)}\right]\left\{\frac{1-\exp\left(d_{2,0}\right)}{d_{2,0}} + \frac{\exp\left(d_{1,0}\right)-1}{d_{1,0}} + \exp\left(d_{2,0}\,\bar{x}\right) - \exp\left(d_{1,0}\,\bar{x}\right)\right\}$$
$$+\frac{\beta_4\,\chi}{\nu}\left\{\frac{\exp\left(d_{1,0}\right)-1}{d_{1,0}} - \exp\left(d_{1,0}\,\bar{x}\right)\right\} \tag{F2}$$

where $d_{1,0} = \frac{\left[b_{1,0}\,Pe_T + \sqrt{b_{1,0}^2\,Pe_T^2 + 4\nu}\right]}{2\chi}$, $d_{2,0} = \frac{\left[b_{1,0}\,Pe_T - \sqrt{b_{1,0}^2\,Pe_T^2 + 4\nu}\right]}{2\chi}$ and $b_{1,0} = \frac{1}{2}\int_{-1}^{1}\bar{u}_{0,0}\,d\bar{y}$

Here, $2b_{1,0}$ represents the volumetric flow rate for patterned electroosmotic flow of a Newtonian fluid given by

$$2b_{1,0} = 2\left[1-\frac{\tanh\left(\bar{\kappa}_0\right)}{\bar{\kappa}_0}\right]\left\{\alpha_1+\alpha_2\frac{\sin(\omega)}{\omega}\right\} \tag{F3}$$

G: Results in the thin EDL limit

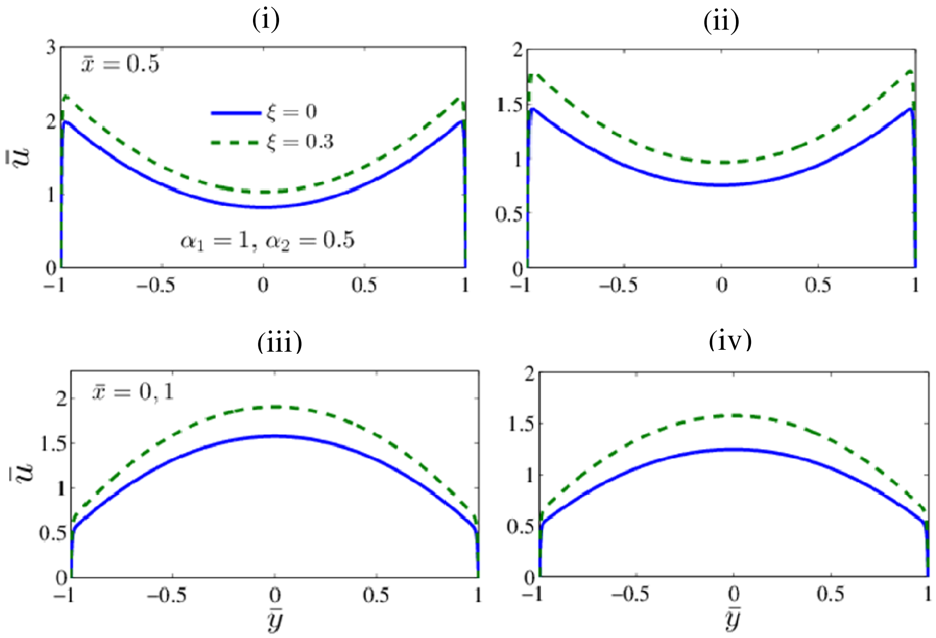

Figure. 2a. Velocity profile in the *y*-direction, evaluated at $\alpha_1 = 1,\ \alpha_2 = 0.5,\ \omega = 2\pi$. (i), (iii) viscoelastic fluid $(De = 0.5)$ and (ii), (iv) Newtonian fluid $(De = 0)$.

When the thickness of the EDL becomes very small (of the order of the few nanometers), the region of excess charge distribution is very less as compared to the channel dimension. In presence of axially modulated surface potential, the favorable pressure gradient for $\omega = 2\pi$ occurs at the middle of the channel while adverse pressure appears to be present at the channel ends the effect of which is clearly reflected in the velocity distribution of Fig. 2a. Similarly, the distribution of the dispersion coefficient shows bimodal behaviour with maximum augmentation occurring in the middle and minimum at the two ends. Also, the degree of fluctuation gets amplified on imposition of non-isothermal condition (ξ) as well as fluid viscoelasticity (De). (as shown in Fig. 2b).

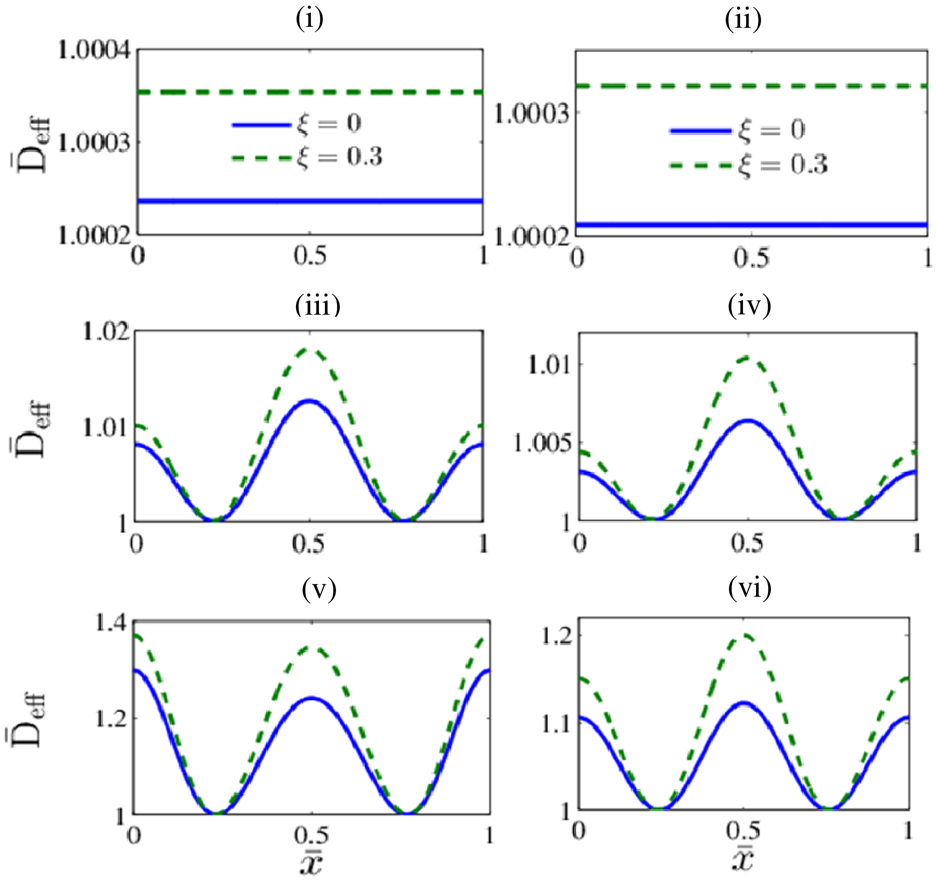

Figure. 2b. The effect of ω on the axial variation of $\overline{D}_{eff}$, evaluated for different values of ξ. (i), (iii) viscoelastic fluid $(De = 0.5)$ and (ii), (iv) Newtonian fluid $(De = 0)$.

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