Electronic supplementary material for

Parent-preferred dispersal promotes cooperation in structured populations

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1. Results for the original birth-death rule

In figure S1, we report the fraction of cooperators as a function of the benefit-to-cost ratio b/c in the three different types of population structures under the birth-death rule with random dispersal mode under which the offspring replaces a random neighbour of the parent. We observe that under the original birth-death rule, cooperators cannot survive in random-regular and small-world networks, even if the benefit-to-cost is high. Furthermore, in scale-free networks just a few cooperators survive when the benefit-to-cost ratio is high. These results show that the original birth-death rule greatly suppresses the evolution of cooperation in social networks [1], even if initially half of the nodes are occupied by cooperators and the benefit-to-cost ratio is high.

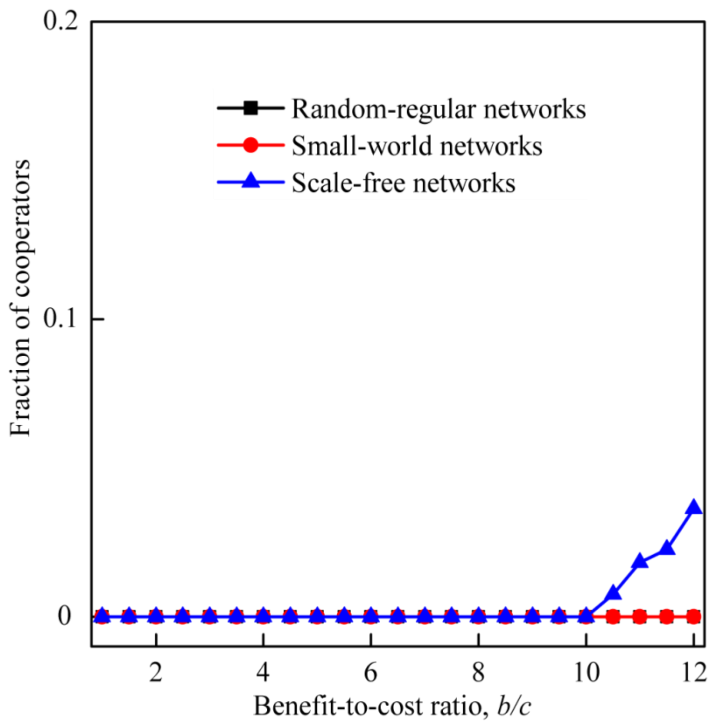

Figure S1: Fraction of cooperators as a function of the benefit-to-cost ratio b/c without offspring-preferred or parent-preferred dispersal, that is, $\alpha = \beta = 0$. Under the birth-death update rule with random dispersal mode, it is extremely difficult for cooperation to evolve in different types of population structures. However, for large benefit-to-cost ratio just a few cooperators can survive in scale-free networks. Here, the intensity of selection w is fixed at 0.01 and initially each individual is designated either as a cooperator or a defector with equal probability.

2. Pair-approximation method for birth-death rule with parent-preferred and offspring-preferred dispersal

By assuming that cooperators are distributed uniformly among the network, we can use the pair-approximation method to theoretically study the evolutionary dynamics of cooperation on regular graphs [1]. Let p_C and p_D respectively denote the fractions of cooperators and defectors in the population, so that $p_C + p_D = 1$. Let p_{CC}, p_{CD}, p_{DC}, and p_{DD} respectively denote the factions of CC, CD, DC, and DD links. We have $p_{CC} + p_{CD} + p_{DC} + p_{DD} = 1$ and $p_{CD} = p_{DC}$. Let $q_{X|Y}$ denote the conditional probability to find an X-player given that the adjacent node is occupied by a Y-player. Here, both X and Y stand for C or D. Then we have $p_{XY} = q_{X|Y} p_Y$ and $q_{C|X} + q_{D|X} = 1$. As a result, based on the relationships of the quantities, we can see that the dynamics of the system can be described by only two quantities, p_C and p_{CC}.

Now, let us consider the probability that a player is selected for reproduction in a regular graph with the number of neighbours k. We know that the fitness of a cooperator who has k_C cooperative neighbours and $k_D = k - k_C$ defective neighbours is $f_C = e^{w(k_C M_{11} + k_D M_{12})}$, and hence the probability that such cooperators are selected for reproduction is proportional to

$$A = p_C \binom{k}{k_C} q_{C|C}^{k_C} q_{D|C}^{k_D} f_C. \qquad (1)$$

Similarly, the fitness of a defector who has k_C cooperative neighbours and k_D defective neighbours is $f_D = e^{w(k_C M_{21} + k_D M_{22})}$, and hence the probability that such defectors are selected for reproduction is proportional to

$$B = p_D \binom{k}{k_C} q_{C|D}^{k_C} q_{D|D}^{k_D} f_D. \qquad (2)$$

Next, we consider the probability that a player's offspring chooses a neighbour to replace. During the replacment process, the parent can explore the expected fitness in four different situations. In the first situation when a cooperator's offspring replaces a cooperative neighbour we have

$$f_{P(C\to C)} = e^{w(k_C M_{11} + k_D M_{12})},$$

and

$$f_{O(C\to C)} = e^{w[M_{11} + (k-1)q_{C|C} M_{11} + (k-1)q_{D|C} M_{12}]},$$

respectively representing the expected fitness of the cooperative parent and its offspring.

In the second situation when a cooperator's offspring replaces a defective neighbour, we have

$$f_{P(C\to D)} = e^{w[(k_C+1)M_{11} + (k_D-1)M_{12}]},$$

and

$$f_{O(C\to D)} = e^{w[M_{11} + (k-1)q_{C|D} M_{11} + (k-1)q_{D|D} M_{12}]},$$

respectively represent the expected fitness of the cooperative parent and its offspring.

In the third situation when a defector's offspring replaces a cooperative neighbour, we have

$$f_{P(D\to C)} = e^{w[(k_C-1)M_{21} + (k_D+1)M_{22}]},$$

and

$$f_{O(D\to C)} = e^{w[(k-1)q_{C|C} M_{21} + (k-1)q_{D|C} M_{22} + M_{22}]},$$

respectively represent the expected fitness of the defective parent and its offspring.

In the fourth situation when a defector's offspring replaces a defective neighbour, we have

$$f_{P(D\to D)} = e^{w(k_C M_{21} + k_D M_{22})},$$

and

$$f_{O(D\to D)} = e^{w[(k-1)q_{C|D} M_{21} + (k-1)q_{D|D} M_{22} + M_{22}]},$$

respectively represent the expected fitness of the defective parent and its offspring.

Based on the information from the four different situations, the probability that a cooperator's offspring chooses to replace a defective neighbour is given by

$$\frac{k_D f_{O(C\to D)}^{\alpha} f_{P(C\to D)}^{\beta}}{k_D f_{O(C\to D)}^{\alpha} f_{P(C\to D)}^{\beta} + k_C f_{O(C\to C)}^{\alpha} f_{P(C\to C)}^{\beta}}.$$

As a result, p_C increases by $1/N$ with probability

$$\mathrm{Prob}(\triangle p_C = \frac{1}{N}) = \sum_{k_C+k_D=k} A \frac{k_D f_{O(C\to D)}^{\alpha} f_{P(C\to D)}^{\beta}}{k_D f_{O(C\to D)}^{\alpha} f_{P(C\to D)}^{\beta} + k_C f_{O(C\to C)}^{\alpha} f_{P(C\to C)}^{\beta}}.$$

Correspondingly, the number of CC pairs increases by $1 + q_{C|D}(k-1)$ and p_{CC} increases by $2[1 + q_{C|D}(k-1)]/(kN)$ with probability

$$\mathrm{Prob}(\triangle p_{CC} = \frac{2[1 + q_{C|D}(k-1)]}{kN}) = A \frac{k_D f_{O(C\to D)}^{\alpha} f_{P(C\to D)}^{\beta}}{k_D f_{O(C\to D)}^{\alpha} f_{P(C\to D)}^{\beta} + k_C f_{O(C\to C)}^{\alpha} f_{P(C\to C)}^{\beta}}.$$

Similarly, the probability that a defector's offspring chooses to replace a cooperative neighbour is given by

$$\frac{k_C f_{O(D\to C)}^{\alpha} f_{P(D\to C)}^{\beta}}{k_C f_{O(D\to C)}^{\alpha} f_{P(D\to C)}^{\beta} + k_D f_{O(D\to D)}^{\alpha} f_{P(D\to D)}^{\beta}}.$$

As a result, p_C decreases by $1/N$ with probability

$$\mathrm{Prob}\left(\triangle p_C = -\frac{1}{N}\right) = \sum_{k_C+k_D=k} B \frac{k_C f_{O(D\to C)}^{\alpha} f_{P(D\to C)}^{\beta}}{k_C f_{O(D\to C)}^{\alpha} f_{P(D\to C)}^{\beta} + k_D f_{O(D\to D)}^{\alpha} f_{P(D\to D)}^{\beta}}.$$

Correspondingly, the number of CC pairs decreases by $(k-1)q_{C|C}$ and p_{CC} decreases by $2(k-1)q_{C|C}/(kN)$ with probability

$$\mathrm{Prob}\left(\triangle p_{CC} = -\frac{2(k-1)q_{C|C}}{kN}\right) = B \frac{k_C f_{O(D\to C)}^{\alpha} f_{P(D\to C)}^{\beta}}{k_C f_{O(D\to C)}^{\alpha} f_{P(D\to C)}^{\beta} + k_D f_{O(D\to D)}^{\alpha} f_{P(D\to D)}^{\beta}}.$$

Combining the two probability calculations above, we have

$$\begin{aligned} \dot{p}_C &= \frac{1}{N} \cdot \mathrm{Prob}\left(\triangle p_C = \frac{1}{N}\right) - \frac{1}{N} \cdot \mathrm{Prob}\left(\triangle p_C = -\frac{1}{N}\right) \\ &= \frac{1}{N} \sum_{k_C=0}^{k} A \frac{k_D f_{O(C\to D)}^{\alpha} f_{P(C\to D)}^{\beta}}{k_D f_{O(C\to D)}^{\alpha} f_{P(C\to D)}^{\beta} + k_C f_{O(C\to C)}^{\alpha} f_{P(C\to C)}^{\beta}} \\ &- \frac{1}{N} \sum_{k_C=0}^{k} B \frac{k_C f_{O(D\to C)}^{\alpha} f_{P(D\to C)}^{\beta}}{k_C f_{O(D\to C)}^{\alpha} f_{P(D\to C)}^{\beta} + k_D f_{O(D\to D)}^{\alpha} f_{P(D\to D)}^{\beta}}, \end{aligned} \tag{3}$$

and

$$\begin{aligned} \dot{p}_{CC} &= \sum_{k_C+k_D=k} \frac{2[1 + q_{C|D}(k-1)]}{kN} \mathrm{Prob}(\triangle p_{CC} = \frac{2[1 + q_{C|D}(k-1)]}{kN}) \\ &- \sum_{k_C+k_D=k} \frac{2(k-1)q_{C|C}}{kN} \mathrm{Prob}(\triangle p_{CC} = -\frac{2(k-1)q_{C|C}}{kN}) \end{aligned}$$

$$= \sum_{k_C=0}^{k} \frac{2[1+q_{C|D}(k-1)]}{kN} A \frac{k_D f^{\alpha}_{O(C\to D)} f^{\beta}_{P(C\to D)}}{k_D f^{\alpha}_{O(C\to D)} f^{\beta}_{P(C\to D)} + k_C f^{\alpha}_{O(C\to C)} f^{\beta}_{P(C\to C)}}$$
$$- \sum_{k_C=0}^{k} \frac{2(k-1)q_{C|C}}{kN} B \frac{k_C f^{\alpha}_{O(D\to C)} f^{\beta}_{P(D\to C)}}{k_C f^{\alpha}_{O(D\to C)} f^{\beta}_{P(D\to C)} + k_D f^{\alpha}_{O(D\to D)} f^{\beta}_{P(D\to D)}}. \quad (4)$$

After the variable substitution, we use numerical integration to solve the two differential equations above for p_C. Figure 2 shows the cooperation level depending on offspring-preferred dispersal strength α and parent-preferred dispersal strength β as predicted by this pair-approximation method.

3. Death-birth update rule with parent-preferred and offspring-preferred dispersal

Initially, each player x is designated to play either C or D, and occupies one site of the network. At each time step, each player x engages in pairwise interactions with all its adjacent neighbours, and then collects its payoff P_x based on the payoff matrix parameters. Furthermore, player x obtains its fitness associated with the payoff information, given as

$$f_x = e^{wP_x},$$

where w is the intensity of selection.

After playing the games, a random individual x is chosen to die, and subsequently the neighbours compete for the empty site. The probability that each neighbour y competes for the empty site is defined as

$$d_{y(x)} = \frac{f^{\alpha}_{x(y\to x)} f^{\beta}_{y(y\to x)}}{\sum_{z\in\Omega_y} f^{\alpha}_{z(y\to z)} f^{\beta}_{y(y\to z)}},$$

where the sum in the denominator is over all the neighbours of y (Ω_y is the set of neighbours of individual y), $f_{z(y\to z)}$ denotes the expected fitness of individual z when the offspring of individual y occupies the site of neighbour z, and $f_{y(y\to z)}$ denotes the expected fitness of individual y when the offspring of individual y occupies the site of neighbour z. Here, α ($\alpha > 0$) represents the offspring-preferred dispersal strength, β ($\beta > 0$) represents the parent-preferred dispersal strength, and $d_{y(x)}$ characterizes the desirability strength of the offspring of individual y to move into the site of individual x.

Based on the neighbour's desirability strength, the probability that individual y

can occupy the site of individual x is

$$P_{y\to x} = \frac{d_{y(x)} f_y}{\sum_{i\in\Omega_x} d_{i(x)} f_i},$$

where the sum in the denominator is over all the neighbours of x (Ω_x is the set of neighbours of individual x), and f_i denotes the obtained fitness of individual i from the interactions with its current neighbours. Here, $d_{i(x)} \times f_i$ represents the competition strength of individual i for the site of individual x. And when $\alpha = 0$ and $\beta = 0$, this rule recovers the original death-birth update rule [2].

4. Imitation update rule with parent-preferred and offspring-preferred dispersal

Initially, each player is designated to play either C or D, and occupies one site of the network. At each time step, each player x engages in pairwise interactions with all its adjacent neighbours, and then collects its payoff P_x based on the payoff matrix parameters. Furthermore, player x obtains its fitness associated with the payoff information, given as

$$f_x = e^{wP_x},$$

where w is the intensity of selection.

After playing the games, a random individual x is chosen to evaluate its strategy. It will either stay with its own strategy or imitate a neighbour's strategy. The probability that each neighbour y enforces its strategy to individual x is defined as

$$d_{y(x)} = \frac{f_{x(y\to x)}^{\alpha} f_{y(y\to x)}^{\beta}}{\sum_{z\in\Omega_y} f_{z(y\to z)}^{\alpha} f_{y(y\to z)}^{\beta}},$$

where the sum in the denominator is over all the neighbours of individual y (Ω_y is the set of neighbours of individual y including individual y), $f_{z(y\to z)}$ denotes the expected fitness of individual z when the individual y enforces its strategy to individual z successfully, and $f_{y(y\to z)}$ denotes the expected fitness of individual y when individual y enforces its strategy to individual z successfully. Here, α ($\alpha > 0$) represents the offspring-preferred dispersal strength, β ($\beta > 0$) represents the parent-preferred dispersal strength, and $d_{y(x)}$ characterizes the desirability strength of individual y to enforce its strategy to individual x.

Based on the neighbour's desirability strength, the probability that the neighbour y can enforce its strategy to individual x is

$$P_{y\to x} = \frac{d_{y(x)} f_y}{\sum_{i\in\Omega_x} d_{i(x)} f_i},$$

where the sum in the denominator is over all the neighbours of x (Ω_x is the set of neighbours of individual x including individual x itself), f_i denotes the obtained fitness of individual i from the interactions with its current neighbours. Here, $d_{i(x)} \times f_i$ represents the competition strength of individual i to enforce its strategy to individual x. And when $\alpha = 0$ and $\beta = 0$, this rule recovers the original imitation update rule [2].

5. Pairwise comparison update rule with parent-preferred and offspring-preferred dispersal

Initially, each player is designated to play either C or D, and occupies one site of the network. At each time step, each player x engages in pairwise interactions with all its adjacent neighbours, and then collects its payoff P_x based on the payoff matrix parameters. Furthermore, player x obtains its fitness associated with the payoff information, given as

$$f_x = e^{wP_x},$$

where w is the intensity of selection.

After playing the games, a random individual x is chosen to evaluate its strategy, and one of the neighbours y is chosen to teach individual x proportion to the neighbour's desirability. Here, we define neighbour y's desirability as

$$d_{y(x)} = \frac{f^{\alpha}_{x(y\to x)} f^{\beta}_{y(y\to x)}}{\sum_{z\in\Omega_y} f^{\alpha}_{z(y\to z)} f^{\beta}_{y(y\to z)}},$$

where the sum in the denominator is over all the neighbours of individual y (Ω_y is the set of neighbours of individual y), $f_{z(y\to z)}$ denotes the expected fitness of individual z when individual y enforces its strategy to individual z successfully, and $f_{y(y\to z)}$ denotes the expected fitness of individual y when individual y enforces its strategy to individual z successfully. Here, α ($\alpha > 0$) represents the offspring-preferred dispersal strength, and β ($\beta > 0$) represents the parent-preferred dispersal strength.

After the neighbour y is chosen, player x adopts individual y's strategy with a probability depending on the fitness difference as

$$P_{y\to x} = \frac{1}{1+\exp[(f_x - f_y)/\kappa]},$$

where κ is the uncertainty by strategy adoptions. Without losing generality, we set $\kappa = 1.0$ in this study. And when $\alpha = 0$ and $\beta = 0$, this rule recovers the original pairwise comparison update rule [2].

6. Results for the three other update rules with parent-preferred and offspring-preferred dispersal

In figure S2, we report how the fraction of cooperators depends on the offspring-preferred dispersal strength α and the parent-preferred dispersal strength β for three other update rules (death-birth, imitation, and pairwise comparison update rules), when parent-preferred and offspring-preferred dispersal modes are considered. We still find that under these three different update rules the parent-preferred dispersal way can promote the evolution of cooperation in different types of population structures, and in scale-free networks cooperation can be enhanced for intermediate offspring-preferred dispersal strength when the parent-preferred dispersal strength is not high. These results show that our main results about effects of parent-preferred and offspring-preferred dispersal are robust against the considered changes of the update rules.

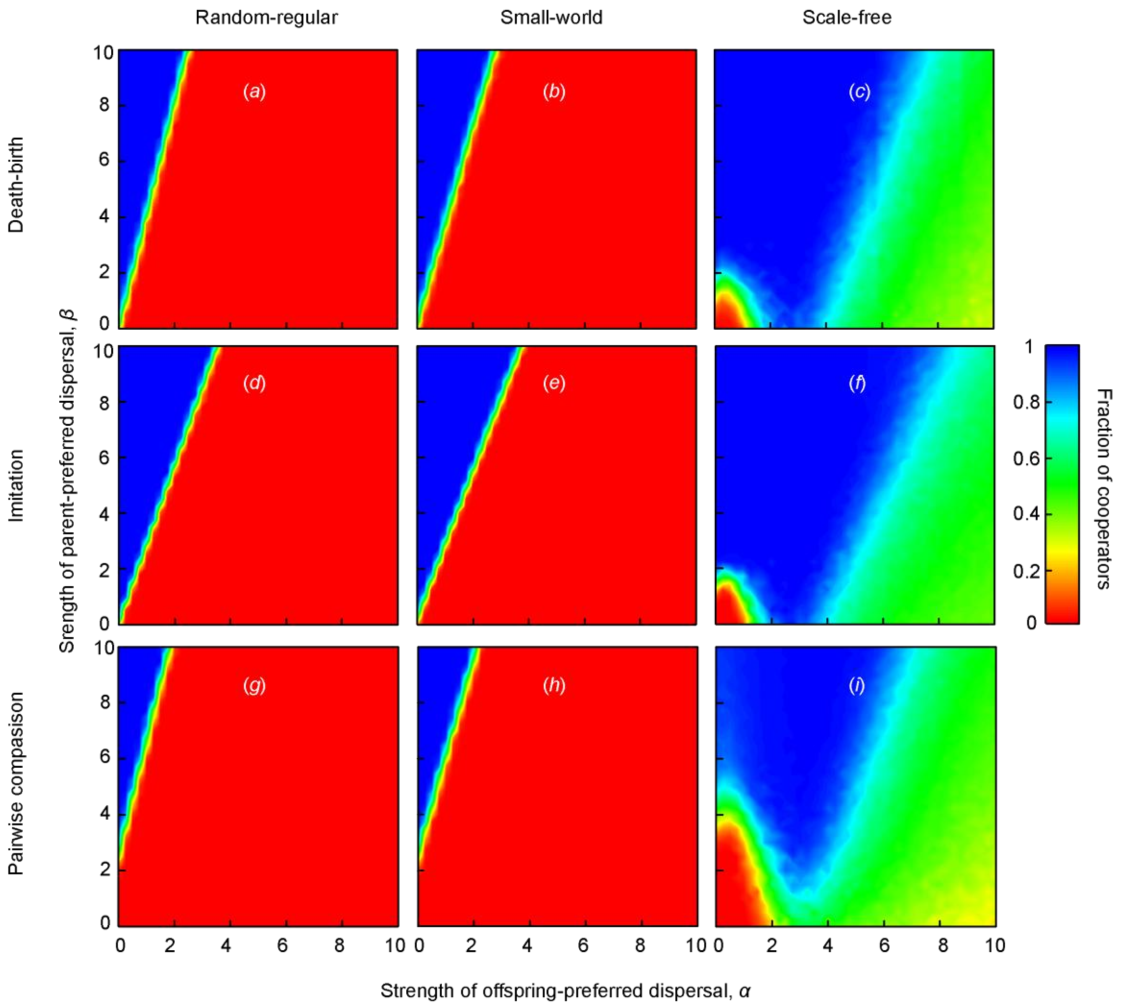

Figure S2: Impact of the strengths of parent-preferred and offspring-preferred dispersal on the fraction of cooperators for different update rules and population structures. Top row depicts the fraction of cooperators depending on offspring-preferred dispersal strength α and parent-preferred dispersal strength β in a contour plot form under death-birth update rule in random-regular (a), small-world (b), and scale-free (c) networks, respectively. Middle row depicts the fraction of cooperators depending on offspring-preferred dispersal strength α and parent-preferred dispersal strength β in a contour plot form under imitation rule in random-regular (d), small-world (e), and scale-free (f) networks, respectively. Bottom row depicts the fraction of cooperators depending on offspring-preferred dispersal strength α and parent-preferred dispersal strength β in a contour plot form under pairwise comparison rule in random-regular (g), small-world (h), and scale-free (i) networks, respectively. Here, $w = 0.01$ and $b/c = 4$. Initially each individual is designated either as a cooperator or a defector with equal probability, and under the pairwise comparison update rule $\kappa = 1.0$.

References

[1] Ohtsuki H, Hauert C, Lieberman E, Nowak MA. 2006 A simple rule for the evolution of cooperation on graphs and social networks. *Nature* **441**, 502-505. (doi:10.1038/nature04605)

[2] Ohtsuki H, Nowak MA. 2006 The replicator equation on graphs. *J. Theor. Biol.* **243**, 86-97. (10.1016/j.jtbi.2006.06.004)