**Supplementary Material C**

**Appendix C.**

In this Appendix, we derive the critical applied pressure for the long-term response of a poro-hyperelastic cylindrical shell for which the parameter . Only limit-point instability will be considered and thus the formulation of the problem is radially symmetric. Since the fluid pressure in the long-term becomes equal to the applied pressure, , the given problem is equivalent to the problem for an ordinary hyperelastic shell, without fluid, which is traction free on the surface  and loaded by tensile stress  on the surface 

, . (C.1)

The effective stress now becomes equal to the total stress. This equivalent problem will be studied here.

Recall the equilibrium equation (3.11)

. [3.11]

The difference between the stresses can be found according to (3.12)

. [3.12]

Consequently, using  and differentiating with respect to  instead of , we obtain from (3.11)

, (C.2)

where

. (C.3)

Note that  is the only stabilizing term in the equilibrium equation (C.2). If , then  and it follows that the radial stress is a constant. Then, because of zero traction boundary conditions at , the applied stress at  must also be zero and the behavior of the shell is thus unstable. On the other hand, if , then . Thus, given zero traction boundary conditions at , the applied stress at  could be an arbitrarily large positive number and the behavior will be stable.

Hence, to judge the stability of the system, it is sufficient to obtain an asymptotic estimate of  for large values of the stretch . First, we represent  as

, (C.4)

where  denotes the terms that contain powers of order smaller than  and  is a second constant to be found. Then, it follows that

, . (C.5)

We assume that

. (C.6)

Now we substitute the derivative  into the equilibrium equation (3.16) for the specific case where  and obtain

 (C.7)

Now we represent the products of the two stretches as



and collect in (C.7) the terms with the highest power order of  to obtain

. (C.8)

By equating powers of  in the last relationship, we obtain a linear equation with respect to 

,

the solution of which is given by

. (C.9)

The constant  can now be found from (C.8) by solving the equation

.

Therefore,

. (C.10)

Hence

 as . (C.11)

We can now obtain the asymptotic estimate for . From (C.3)

. (C.12)

But

. (C.13)

After integration of (C.13) and taking into account zero traction boundary condition at , we obtain

. (C.14)

Therefore, the stress at the surface  can be found as

. (C.15)

In the original problem for the poro-hyperelastic shell, at  the radial stress  and thus the applied pressure becomes

. (C.16)

This is the maximum pressure that can be applied to the poro-hyperelastic shell with  if we take into account only the radially symmetric mode of stability loss. This result in fact does not depend on the value of the fluid pressure and remains valid as long as the fluid pressure is uniform across the thickness of the shell (including zero fluid pressure).