Supplementary material from

Avoiding/inducing dynamic buckling in a thermomechanically coupled plate: a local and global analysis of slow/fast response

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Dimensional reduced equations of motion and expression of the relevant coefficients

- Dimensional reduced coupled nonlinear equations of motion [2]:

$$\begin{aligned}
&a_{11}\ddot{W} + a_{12}\dot{W} + (a_{13} + a_{14})W + a_{15}W^3 \\
&+ a_{16}T_{R1} + a_{17}W\,T_{R0} = a_{18}F_3^{(0)} \\
&(a_{21} + a_{22})\dot{T}_{R0} + a_{23}T_{R0} + a_{24}\dot{W}\,W + a_{25}T_\infty = a_{26}E^{(0)} \\
&a_{31}\dot{T}_{R1} + a_{32}T_{R1} + a_{33}\dot{W} = a_{34}E^{(1)}
\end{aligned} \tag{A.1}$$

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- Coefficients a_{ij} of Eq. (A.1) for an orthotropic single layer plate:

$$a_{11} = -\frac{1}{4}abh\rho;\; a_{12} = -\frac{1}{4}ab\delta;$$

$$a_{13} = \frac{-\pi^4 h^3 (a^2(a^2 Q_{22} + 2b^2(Q_{12} + 2Q_{66})) + b^4 Q_{11}}{48a^3b^3};$$

$$a_{14} = \frac{\pi^2(b^2 p_x + a^2 p_y)}{4ab};$$

$$a_{15} = \frac{-\pi^4 h(Q_{11}Q_{22} - Q_{12}^2)(a^4 Q_{22} + b^4 Q_{11})}{64a^3b^3Q_{11}Q_{22}};$$

$$a_{16} = \frac{\pi^2 h^3(a^2\beta_{22} + b^2\beta_{11})}{48ab};$$

$$a_{17} = (8abhQ_{66}(a^2\beta_{11}Q_{22} + b^2\beta_{22}Q_{11} - Q_{12}(a^2\beta_{22} + b^2\beta_{11})))/(3Q_{66}(a^4Q_{22} - 2a^2b^2Q_{12} + b^4Q_{11}) - 3a^2b^2(Q_{12}^2 - Q_{11}Q_{22}));$$

$$a_{18} = -\frac{4ab}{\pi^2};\; a_{21} = -\frac{1}{4}abhc_\epsilon;$$

$$a_{22} = (abhT_{ref}(a^2\beta_{22}b^2(2\beta_{11}Q_{12} - \beta_{22}Q_{11}) - a^2\beta_{11}^2b^2Q_{22} - Q_{66}(b^2\beta_{11} - a^2\beta_{22})^2))/(4Q_{66}(a^4Q_{22} - 2a^2b^2Q_{12} + b^4Q_{11}) - 4a^2b^2(Q_{12}^2 - Q_{11}Q_{22}));$$

$$a_{23} = \frac{-\pi^2 h(a^2\lambda_{22} + b^2\lambda_{11})}{4ab} - \frac{1}{2}abH;\; a_{24} = \frac{8abH}{\pi^2};$$

$$a_{25} = \frac{hT_{ref}(\beta_{11}(\frac{a^2Q_{12}}{Q_{11}} - b^2) + \beta_{22}(\frac{b^2Q_{12}}{Q_{22}} - a^2))}{3ab};$$

$$a_{26} = -\frac{4ab}{\pi^2};\; a_{31} = -\frac{1}{48}abh^3c_\epsilon;$$

$$a_{32} = \frac{-h(a^2(6b^2(hH + 2\lambda_{33}) + \pi^2h^2\lambda_{22}) + \pi^2b^2h^2\lambda_{11})}{48ab};$$

$$a_{33} = \frac{-\pi^2h^3T_{ref}(a^2\beta_{22} + b^2\beta_{11})}{48ab};\; a_{34} = -\frac{4ab}{\pi^2}$$

in terms of the following lamina properties:

$Q_{11}, Q_{22}, Q_{12}, Q_{66}:$ stress-reduced elastic stiffnesses [37]

$Q_{11} = \dfrac{Y_1}{1-\nu_{12}\nu_{21}}\ Q_{22} = \dfrac{Y_2}{1-\nu_{12}\nu_{21}}, Q_{12} = \dfrac{\nu_{12}Y_2}{1-\nu_{12}\nu_{21}}, Q_{66} = G_{12}$

$Y_1, Y_2, G_{12}:$ longitudinal modulus of rigidity in x and y direction and shear modulus

$\nu_{12}:$ Poisson's ratio

$\rho:$ mass density

$\delta:$ damping coefficient

$\lambda_{11}, \lambda_{22}, \lambda_{33}:$ thermal conductivities along the x, y and z directions

$\beta_{11}, \beta_{22}:$ thermoelastic stiffnesses

$$\beta_{11} = Q_{11}\alpha_1 + Q_{12}\alpha_2,\ \beta_{22} = Q_{12}\alpha_1 + Q_{22}\alpha_2$$

$\alpha_1, \alpha_2:$ thermal expansions along x and y directions

$C_\epsilon:$ thermal capacity $C_\epsilon = \rho c_v$

$c_v:$ specific heat at constant strain

$H:$ boundary conductance

$a, b, h:$ length, width and thickness of the rectangular plate

Nondimensional reduced equations of motion and expression of the relevant coefficients

- Nondimensional coefficients $\bar{a}_{ij}$ of Eqs. (2.1) in the relevant paper, herein reported for convenience:

$$\begin{aligned}
&\ddot{W} + \bar{a}_{12}\dot{W} + \bar{a}_{13}W + \bar{a}_1 W^3 + \bar{a}_1 T_{R1} + \bar{a}_{16} W\, T_{R0} + \bar{a}_{17}\cos t = 0\\
&\dot{T}_{R0} + \bar{a}_{22}T_{R0} + \bar{a}_{23}\alpha_1 T_\infty + \bar{a}_{24}\dot{W}\, W + \bar{a}_{25}e_0(t) = 0\\
&\dot{T}_{R1} + \bar{a}_{32}T_{R1} + \bar{a}_{33}\dot{W} + \bar{a}_{34}e_1(t) = 0
\end{aligned} \tag{A.2}$$

$$\bar{a}_{12} = \frac{\delta^*}{\eta};\ \bar{a}_{13} = \frac{1-p}{\eta^2};$$
$$\bar{a}_{14} = \frac{3(a_1 - a_2^2)(a_1 r^4 + 1)}{4a_1\eta^2(a_1 r^4 + 2(a_2 + 2a_3)r^2 + 1)};$$
$$\bar{a}_{15} = \frac{-(\alpha a_2 + 1)(\beta_1 r^2 + 1)}{\pi^2\eta^2(a_1 r^4 + 2(a_2 + 2a_3)r^2 + 1)};$$
$$\begin{aligned}\bar{a}_{16} = {} & (128a_3 r^2(\alpha a_2 + 1)(\beta_1(a_2 r^2 - 1) - a_1 r^2 + a_2)) \\ & /(\pi^4\eta^2(a_1 r^4 + 2(a_2 + 2a_3)r^2 + 1)(r^2(a_1(a_3 r^2 + 1) \\ & - a_2(a_2 + 2a_3)) + a_3));\end{aligned}$$
$$\bar{a}_{17} = -\frac{f}{\eta^2};$$
$$\begin{aligned}\bar{a}_{22} = {} & ((r^2(a_1(a_3 r^2 + 1) - a_2(a_2 + 2a_3)) + a_3)(4\lambda_2 m + \pi^2 s^2 \\ & (\lambda_1 r^2 + 1)))/(\eta n s^2(a_3(\alpha a_2\beta + \beta + 1) + a_3 r^4 \\ & (\beta\beta_1^2(\alpha a_2 + 1) + a_1) + r^2(\beta\beta_1^2(\alpha a_2 + 1) - 2(a_2 + a_3) \\ & \beta\beta_1(\alpha a_2 + 1) + a_1(\alpha a_2\beta + \beta + 1) - a_2(a_2 + 2a_3))));\end{aligned}$$
$$\begin{aligned}\bar{a}_{23} = {} & -(64\lambda_2 m(r^2(a_1(a_3 r^2 + 1) - a_2(a_2 + 2a_3)) + a_3))/(\eta n\pi^2 s^4(a_3 r^4 \\ & (a_1 + \beta\beta_1^2(a_2\alpha + 1)) + r^2(a_1(a_2\alpha\beta + \beta + 1) - 2\beta\beta_1(a_2 + a_3)(a_2\alpha + 1) \\ & - a_2(a_2 + 2a_3) + \beta\beta_1^2(a_2\alpha + 1)) + a_3(a_2\alpha\beta + \beta + 1)));\end{aligned}$$
$$\begin{aligned}\bar{a}_{24} = {} & -(4\beta(r^2(a_1(a_3 r^2 + 1) - a_2(a_2 + 2a_3)) + a_3)(a_2\beta_1 + a_1 \\ & (r^2(a_2 - \beta_1) - 1)))/(3a_1(a_3(\alpha a_2\beta + \beta + 1) + a_3 r^4 \\ & (\beta\beta_1^2(\alpha a_2 + 1) + a_1) + r^2(\beta\beta_1^2(\alpha a_2 + 1) - 2(a_2 + a_3) \\ & \beta\beta_1(\alpha a_2 + 1) + a_1(\alpha a_2\beta + \beta + 1) - a_2(a_2 + 2a_3))));\end{aligned}$$
$$\begin{aligned}\bar{a}_{25} = {} & (r^2(a_2(a_2 + 2a_3) - a_1(a_3 r^2 + 1)) - a_3)/(\eta(a_3 r^4(a_1 \\ & + \beta\beta_1^2(a_2\alpha + 1)) + r^2(a_1(a_2\alpha\beta + \beta + 1) - 2\beta\beta_1(a_2 + a_3)(a_2\alpha + 1) \\ & - a_2(a_2 + 2a_3) + \beta\beta_1^2(a_2\alpha + 1)) + a_3(a_2\alpha\beta + \beta + 1)));\end{aligned}$$
$$\bar{a}_{32} = \frac{12\lambda_2(m+1) + \pi^2 s^2(\lambda_1 r^2 + 1)}{\eta n s^2};$$
$$\bar{a}_{33} = \pi^2\beta(\beta_1 r^2 + 1);$$
$$\bar{a}_{34} = -12/\eta$$

in terms of the following nondimensional parameters:

$$r=\frac{a}{b};\ s=\frac{h}{a};\ a_1=\frac{Q_{22}}{Q_{11}};\ a_2=\frac{Q_{12}}{Q_{11}};\ a_3=\frac{Q_{66}}{Q_{11}};\ \alpha=\frac{\alpha_2}{\alpha_1};\ \beta_1=\frac{\beta_{22}}{\beta_{11}};$$
$$\delta^*=\frac{\delta}{\omega\rho h};\ \lambda_1=\frac{\lambda_{22}}{\lambda_{11}};\lambda_2=\frac{\lambda_{33}}{\lambda_{11}};\ \beta=\frac{\alpha_1 T_{ref}\beta_{11}}{C_\epsilon};\ n=\frac{a^2C_\epsilon}{\lambda_{11}}\omega;$$
$$m=\frac{Hh}{2\lambda_{33}};\ \eta=\frac{\Omega}{\omega};\ p=\frac{\pi^2}{\omega^2\rho h}(\frac{p_x}{a^2}+\frac{p_y}{b^2});\ f=\frac{16F}{\pi^2\omega^2\rho h^2}$$

Physical properties of the material used for numerical simulations

- Physical properties of an epoxy/carbon fiber composite plate:

$$\begin{aligned}&Y_1=1.72\cdot 10^{11}\frac{N}{m^2},\ \nu_{12}=0.25,\ \rho=1940\frac{kg}{m^3},\ \lambda_{11}=36.42\frac{W}{m\cdot K},\\ &\alpha_1=0.57\cdot 10^{-6}\frac{1}{K},\ Y_2=6.91\cdot 10^{9}\frac{N}{m^2},\ G_{12}=3.45\cdot 10^{9}\frac{N}{m^2},\\ &\lambda_{22}=0.96\frac{W}{m\cdot K},\ \alpha_2=35.6\cdot 10^{-6}\frac{1}{K},\ c_v=400\frac{J}{kg\cdot K},\\ &\delta=330\frac{N\cdot s}{m^3},\ H=100\frac{W}{m^2\cdot K}\end{aligned}\tag{A.3}$$

  The values of the material elastic and thermal properties, which are assumed to be independent of the temperature, are taken from [34], except for the specific heat, c_v, which is given a lower value in order to activate thermal processes with no computational criticalities due to too low thermal stiffnesses, while also working with acceptable values of the thermal excitations.

- Nondimensional numerical coefficients of Eqs. (2.1) in the paper (here Eqs. (A.2)) for an epoxy/carbon fiber composite plate (A.3) of dimensions $a=b=1\,m$ and $h=0.01\,m$, used for the numerical analyses:

$$\begin{aligned}&\bar{a}_{12}=0.0592,\ \bar{a}_{13}=(1-p),\ \bar{a}_{14}=0.6827,\ \bar{a}_{15}=-0.3674,\ \bar{a}_{16}=-0.9658,\\ &\bar{a}_{17}=-f,\ \bar{a}_{22}=9.1137\cdot 10^{-5},\ \bar{a}_{23}=-1.4507,\ \bar{a}_{24}=1.01\cdot 10^{-4},\\ &\bar{a}_{25}=-0.997719,\ \bar{a}_{32}=7.8735\cdot 10^{-4},\ \bar{a}_{33}=8.8714\cdot 10^{-4},\ \bar{a}_{34}=-12\end{aligned}$$